Informal Risk Sharing with Local Information*

Attila Ambrus[†], Wayne Y. Gao[‡], Pau Milán[§]

June 9, 2020

Abstract

This paper considers the effect of contracting limitations in risk-sharing networks, arising for example from observability, verifiability, complexity or cultural constraints. We derive necessary and sufficient conditions for Pareto efficiency under these constraints in a general setting, and we provide an explicit characterization of Pareto efficient bilateral transfer profiles under CARA utility and normally distributed endowments. Our model predicts that network centrality is positively correlated with consumption volatility, as more central agents become quasi-insurance providers to more peripheral agents. The proposed framework has important implications for the empirical specification of risk-sharing tests, allowing for local risk-sharing groups that overlap within the village network.

Keywords: social network, risk sharing, Pareto efficiency, local information

*We thank Dirk Bergemann, Francis Bloch, Vasco Carvalho, Jan Eeckhout, Johannes Gierlinger, Joan de Martí, Marcel Fafchamps, Dean Karlan, Larry Samuelson, Todd Sarver and Christopher Udry for very useful comments. We thank Weiting Miao for excellent research assistance.

[†]Attila Ambrus: Department of Economics, Duke University, Durham, NC 27708, USA; attila.ambrus@duke.edu.

[‡]Wayne Y. Gao: Department of Economics, University of Pennsylvania, Philadelphia, PA 19104, USA; waynegao@upenn.edu.

[§]Pau Milán: Department of Economics, Universitat Autònoma de Barcelona, Edifici B, Bellaterra 08193, Spain; pau.milan@gmail.com.

Contents

1 Introduction

Informal insurance arrangements in social networks have been shown to play an important role at smoothing consumption in a number of different contexts (Ellsworth 1988, Rosenzweig 1988, Deaton 1992, Paxson 1993, Udry 1994, Townsend 1994, Grimard 1997, Fafchamps and Lund 2003, and Fafchamps and Gubert 2007). A main finding in this literature is that informal insurance achieves imperfect consumption smoothing.[1] There are different theoretical explanations as to why perfect risk sharing is not possible. One leading explanation is the presence of enforcement constraints: since risk-sharing arrangements are informal, they have to satisfy incentive compatibility, implying an upper bound on the amount of transfer that individuals can credibly promise to each other. This type of explanation has been explored in a social network framework by Ambrus, Mobius, and Szeidl (2014).[2]

In this paper we explore an alternative explanation featuring imperfectness of the contracting environment. Specifically, we assume that bilateral risk sharing arrangements between a pair of agents cannot be made contingent on everyone's endowment realizations in the community (*global information*), but only on a pair specific subset of endowment realizations (*local information*). These contractibility restrictions can come from limited observability or verifiability of endowment realizations of agents located far enough on the social network, social norms and complexity costs on writing contracts, among other sources. The empirical relevance of local information is documented by Alatas et al. (2016), who find that households' information about each others' financial situations is negatively correlated with the social distance between them.

In most of this paper, for expositional purposes, we focus on the specification where each individual in a network can only observe her own and her neighbors' endowment realizations, and the local information each linked pair can contract upon consists only of the endowment realizations that they can commonly observe - i.e., their own and their common neighbors' endowment realizations. However, we show how the results extend to more general contracting environments, where the informational network, that describes what information each contract can condition on, is defined independently of the physical transfer network, through which real transfers (monetary, in-kind, or otherwise) occur.

[1] Some recent papers, like Schulhofer-Wohl (2011) and Mazzocco and Saini (2012) point out that in some contexts perfect risk-sharing cannot be rejected when allowing for heterogeneous preferences. We discuss how our work is related to this literature later in this section. On the other extreme, Kazianga and Udry (2006) find a setting in which informal social insurance does not improve welfare over autarchy.

[2] See also Karlan et al. (2009), who investigate enforcement constraints in the case of a single borrowing transaction. There is also an extensive literature on the effects of limited commitment on risk-sharing possibilities for a pair of individuals instead of general networks (Coate and Ravallion 1993, Kocherlakota 1996, Ligon 1998, Fafchamps 1999, Ligon, Thomas, and Worrall 2002, Dubois, Jullien, and Magnac 2008).

Relative to previous models, our framework provides a number of new and testable predictions. We find that centrally located individuals become quasi insurance providers to more peripheral households.[3] Further, the current setup formalizes, and indeed generalizes, the notion of a "local sharing group" that has been invoked recently in the risk-sharing tests performed in the development literature.

Existing models of informal risk sharing in networks (Bloch et al. 2008, Bramoullé and Kranton 2007, Ambrus, Mobius, and Szeidl 2014, Ambrus and Elliott 2020) assume that any bilateral arrangement between connected individuals can be conditioned on global information, meaning the community's full set of endowment realizations.[4] We find that this explanation generates qualitatively different predictions relative to models of informal insurance with enforcement constraints. Hence, our results can help future empirical work identify which type of constraint plays the key role in maintaining informal insurance arrangements away from efficiency.[5]

There is a line of theoretical literature investigating the effect of imperfect observability of incomes on informal risk sharing arrangements between two individuals in isolation: see for example Townsend (1982), Thomas and Worrall (1990), and Wang (1995). The questions investigated in this literature are fundamentally different from the ones we focus on, mainly because we are interested in questions that are inherently network related.[6]

The current framework also speaks to an ongoing debate in the development literature that emphasizes the importance of appropriately defining individuals' risk-sharing groups in empirical work (Mazzocco and Saini 2012, Angelucci, De Giorgi, and Rasul 2017, Attanasio, Meghir, and Mommaerts 2018, Munshi and Rosenzweig 2016). A general trend in this literature considers alternative sub-groups within communities as the relevant risk-sharing units of individuals (e.g. an individual's sub-caste or extended family). They argue that classical empirical tests of risk sharing (Townsend, 1994) must be adapted to accommodate heterogeneity in individu-

[3]Throughout the paper we maintain the terminology "individuals", even though in many contexts the relevant unit of analysis is households.

[4]Bloch, Genicot, and Ray (2008) consider different types of exogenously-specified transfer rules, but these arrangements can depend on nonlocal information, potentially achieving first-best outcomes. See also Bourlès, Bramoullé, and Perez-Richet (2017), where individuals are motivated to send transfers to their neighbors for explicit altruistic reasons, but bilateral transfers depend on transfers among other individuals. More recently, Bourlès, Bramoullé, and Perez-Richet (2018) extend their altruism model to a risk-sharing environment and find that efficient insurance is possible under certain altruism networks.

[5]Empirical papers trying to distinguish among different reasons of imperfectness of informal insurance contracts include Kinnan (2017) and Karaivanov and Townsend (2014). For an empirical test between full insurance versus informational constraints, see Ligon (1998).

[6]Other differences include that our analysis is static while the above papers are inherently dynamic, and in our paper individuals perfectly observe local information (but not beyond), while in the above papers incomes are not observable even between two connected individuals.

als' risk sharing communities. However, they only allow for a limited form of heterogeneity in which group membership is mutually exclusive and groups do not interact among themselves. Instead, we provide a general framework that can accommodate these "partition models", but also allows for local risk-sharing groups that overlap in complicated ways along a network of local information.

Our paper is also related to the recent line of papers pointing out that allowing for heterogeneous preferences, in some contexts the full insurance hypothesis cannot be rejected, or at least the imperfection of the insurance can be bounded to be small: see Schulhofer-Wohl (2011), Mazzocco and Saini (2012) and Chiappori et al. (2014). In some settings this hinges on some specific type of preference heterogeneity, for example in the context of Chiappori et al. (2014) it requires that wealth and risk preferences are uncorrelated, which is at odds with the standard assumption of decreasing risk aversion.[7] Nevertheless, it is certainly possible that in some context informal social insurance is close to perfect. However, in other contexts empirical research found that informal social insurance is very ineffective and does not improve welfare relative to autarky (see the context in Kazianga and Udry (2006), and for certain types of risks in the context of Goldstein et al. (2001)). There are also some similarities between our work and the above literature. The latter investigates the role of heterogeneity of preferences in informal risk sharing, while our paper focuses on the role of heterogeneity in network positions.

The first part of our analysis characterizes Pareto efficient risk-sharing arrangements under local information constraints for general (concave) and possibly heterogeneous utility functions and endowment distributions. We show that Pareto efficiency in our context (subject to local information constraints) is equivalent to pairwise efficiency, that is the requirement that the risk-sharing agreement between any pair of neighbors is efficient, taking all other agreements between neighbors fixed. This means that any decentralized negotiation procedure that leads to an outcome in which neighbors do not leave money on the table would yield a Pareto efficient risk-sharing arrangement.

In the benchmark model with global information, the necessary and sufficient conditions for Pareto optimality, referred to as the Borch rule (Wilson, 1968; Borch, 1962) can be derived using standard techniques, and they state that the ratios of any two individuals' marginal utilities of consumption must be equalized across states. We can generalize the Borch rule to this setting by showing that a necessary and sufficient condition for Pareto optimality of a risk-sharing arrangement with local information equates the ratios of *expected* marginal utilities of consumption for each linked pair, where expectations are conditional on local states (i.e. on

[7]For a recent paper finding support for preferences exhibiting decreasing risk aversion, see Paravisini, Rappoport, and Ravina (2016).

the realizations of the contractible endowments).

The generalized Borch rule can be used to verify the Pareto efficiency of consumption plans achieved by candidate transfer agreements in concrete specifications of our model. We provide this characterization for the case of CARA utilities and jointly normally distributed endowments in the context when local information of a pair includes the endowment realizations of the pair and common neighbors. The characterization is particularly simple in the setting of independent endowments: each individual shares her endowment realization equally among her neighbors and herself; on top of that, the arrangement can include state independent transfers, affecting the distribution of surplus but not the aggregate welfare.[8]

For the more general case of symmetrically correlated endowment realizations in the CARA-normal setting, we show that efficient risk-sharing can still be achieved by transfers that are linear in endowment realizations and strictly bilateral (i.e. only contingent on the endowment realizations of the pair involved). In contrast to the local equal sharing rule that obtains in the case of independent endowments, we find that if individuals i and j are linked, increasing the exposure of i to transfers from non-common neighbors increases the share of i's endowment realization transferred to j, relative to local equal sharing, and decreases the share of j's endowment realization transferred to i. These correction terms, which are complicated functions of the network structure, take into account that more centrally located individuals are more exposed to the common shock component, and optimally correct for this discrepancy.

We show that more central individuals tend to end up with a higher consumption variance because they serve as quasi insurance providers to more peripheral neighbors. For large random graphs we show this analytically, and for specific village networks from real world data we show it via simulations. For a fixed set of welfare weights, more central individuals are compensated for this service through higher state-independent transfers ("insurance premium"). This is contrary to the predictions from models with enforcement constraints, like Ambrus, Mobius, and Szeidl (2014), in which more centrally connected individuals are better insured (i.e. end up with smaller consumption variance).[9] In the final section we show how our results extend to more general correlation structures on endowments within the CARA-normal framework, and to specifications of the model outside the analytically tractable CARA-normal setting.

[8]This type of transfer arrangement, which we refer to as the local equal sharing rule, is considered as an ad hoc sharing rule in Belhaj and Deroïan (2012), who consider the effect of equal sharing rules on risk-taking behavior of agents, and more recently in Gao and Moon (2016).

[9]In a separate paper, Milán et al. (2018) test the pairwise transfer scheme predicted by local information constraints against the observed food exchanges between Tsimane' households in the Bolivian Amazon. They find that bilateral transfers can be explained by network centrality, as predicted in Proposition 4 below, which provides further supporting evidence for the model.

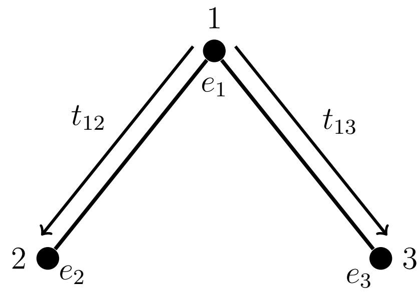


Figure 1: A Simple 3-Individual Network

2 Illustrative Examples

2.1 Basic Setting

Before investigating general network structures, we first consider the simplest non-trivial network, where three individuals, denoted by 1, 2 and 3, are minimally connected in a line. Despite its simplicity, this example provides some useful insights on how local information constraints affect efficient risk-sharing arrangements.

Assume that individuals have homogeneous CARA preferences of the form $u(x) = -\exp(-rx)$, and that endowments $e_1, e_2, e_3 \sim_{iid} \mathcal{N}(0, \sigma^2)$. Only linked individuals may enter into risk-sharing arrangements to mitigate endowment risks. Let t_{12} denote the net ex post transfer from individual 1 to individual 2, and t_{13} the net transfer from individual 1 to individual 3. Let x_1, x_2, x_3 denote the final consumption to individuals after the transfers are implemented, i.e., $x_1 = e_1 - t_{12} - t_{13}$, $x_2 = e_2 + t_{12}$ and $x_3 = e_3 + t_{13}$.

2.2 Global Information

First we consider the benchmark case when bilateral risk-sharing arrangements can be conditioned on global information, that is on all three individuals' endowment realizations: t_{12}, t_{13} can be arbitrary functions of the endowments e_1, e_2, e_3. Standard arguments (see Wilson, 1968) establish that Pareto efficient transfer rules t_{12}, t_{13} are the ones maximizing the social planner's problem:

$$\mathbb{E}\left[\sum_{i=1}^{3} \lambda_i u(x_i)\right] = \mathbb{E}\left[\lambda_1 u(e_1 - t_{12} - t_{13}) + \lambda_2 u(e_2 + t_{12}) + \lambda_3 u(e_3 + t_{13})\right],$$

for some $\lambda_1, \lambda_2, \lambda_3 \in (0,1)$ s.t. $\lambda_1 + \lambda_2 + \lambda_3 = 1$. By the well-known Borch rule (Borch, 1962; Wilson, 1968), the necessary and sufficient conditions for optimality are:

$$\lambda_1 u^{'}(e_1 - t_{12} - t_{13}) = \lambda_2 u^{'}(e_2 + t_{12}) = \lambda_3 u^{'}(e_3 + t_{13}) \ \forall e_1, e_2, e_3.$$

With CARA utility, this yields the *global equal sharing rule*:

$$t_{12}(e_1, e_2, e_3) = \frac{1}{3}e_1 - \frac{2}{3}e_2 + \frac{1}{3}e_3 - \frac{1}{3r}\ln\left(\lambda_2^2/\lambda_1\lambda_3\right)$$

and similarly for t_{13}, leading to the final consumption plan:

$$\begin{cases} x_1 = \frac{1}{3}(e_1 + e_2 + e_3) + \frac{1}{3r}\ln\left(\lambda_1^2/\lambda_2\lambda_3\right), \\ x_2 = \frac{1}{3}(e_1 + e_2 + e_3) + \frac{1}{3r}\ln\left(\lambda_2^2/\lambda_1\lambda_3\right), \\ x_3 = \frac{1}{3}(e_1 + e_2 + e_3) + \frac{1}{3r}\ln\left(\lambda_3^2/\lambda_1\lambda_2\right). \end{cases} \tag{1}$$

That is, Pareto efficient risk-sharing arrangements in every state divide each realized endowment shock equally among all individuals, and the global equal sharing is then corrected by state-independent transfers that implement the welfare weights.

2.3 Local information

Suppose now that each individual's endowment realization is only locally observed by immediate neighbors, so that the transfers t_{12}, t_{13} in the risk-sharing arrangements can be contingent on the endowment realizations that each linked pair of individuals commonly observe, that is, $t_{12} = t_{12}(e_1, e_2)$, $t_{13} = t_{13}(e_1, e_3)$.

It is no longer possible to achieve consumption plans on the Pareto frontier, given by (1), subject to these local information constraints. However, a necessary condition for a transfer arrangement to be socially optimal is that, for any given realization of e_1 and e_2, t_{12} should maximize $\lambda_1 u(e_1 - t_{12} - t_{13}) + \lambda_2 u(e_2 + t_{12})$, given the distribution of e_3 conditional on e_1 and e_2, and the implied distribution of consumption levels (net of t_{12}) induced by t_{13}.[10] In short, given t_{13}, t_{12} should maximize the planner's welfare function:

$$\max_{t_{12}} \int \left[\lambda_1 u(e_1 - t_{12} - t_{13}) + \lambda_2 u(e_2 + t_{12})\right] f_{3|1,2}(e_3)\, de_3$$

The necessary and sufficient FOC for this maximization problem is:

$$\lambda_1 \mathbb{E}\left[u^{'}(e_1 - t_{12} - t_{13}(e_1, e_3)) \middle| e_1, e_2\right] = \lambda_2 u^{'}(e_2 + t_{12}),$$

[10] We show in Section 3 that this condition is actually also sufficient.

and similarly for t_{13} given t_{12}.

Solving this system of two integral equations, we obtain the following transfer rule

$$t_{12}\left(e_1, e_2\right) = \frac{1}{3}e_1 - \frac{1}{2}e_2 - \frac{1}{24}r\sigma^2 - \frac{1}{3r}\ln\left(\lambda_1\lambda_3/\lambda_2^2\right) \tag{2}$$

and similarly for $t_{13}\left(e_1, e_3\right)$. Notice the transfers can be decomposed into three parts. The first part, $\frac{1}{3}e_1 - \frac{1}{2}e_2$, corresponds to the "*local equal sharing rule*", which is the local variant of the equal sharing rule. It implies that individual i's endowment e_i is equally shared by i and i's neighbors, i.e., $t_{ij} = \frac{e_i}{d_i+1} - \frac{e_j}{d_j+1}$. The second part of the equations in (2), $-\frac{1}{24}r\sigma^2$, corresponds to a state-independent transfer that can be regarded as the "insurance premium" paid by the "net insurance purchaser" to the "net insurance provider". In this case, as the final consumption are

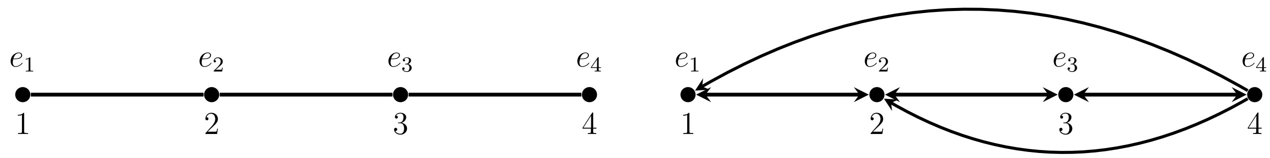
$$\begin{cases} x_1 = \frac{1}{3}e_1 + \frac{1}{2}e_2 + \frac{1}{2}e_3 + \frac{1}{12}r\sigma^2 + \frac{1}{3r}\ln\left(\lambda_1^2/\lambda_2\lambda_3\right), \\ x_2 = \frac{1}{3}e_1 + \frac{1}{2}e_2 - \frac{1}{24}r\sigma^2 + \frac{1}{3r}\ln\left(\lambda_2^2/\lambda_1\lambda_3\right), \\ x_3 = \frac{1}{3}e_1 + \frac{1}{2}e_3 - \frac{1}{24}r\sigma^2 + \frac{1}{3r}\ln\left(\lambda_3^2/\lambda_1\lambda_2\right), \end{cases}$$

individual 1 takes extra risk exposure $\frac{1}{3}e_1 + \frac{1}{2}e_2 + \frac{1}{2}e_3$ in comparison to individuals 2 and 3, $\frac{1}{3}e_1 + \frac{1}{2}e_2$ or $\frac{1}{3}e_1 + \frac{1}{2}e_3$. Hence, individual 1 is rewarded the certainty equivalent (CE) for her intermediary role in risk sharing. The third part of the equations in (2), $-\frac{1}{3r}\ln\left(\lambda_1\lambda_2/\lambda_3^2\right)$, redistributes wealth according to the welfare weights assigned to different individuals (it is zero when $\lambda_1 = \lambda_2 = \lambda_3$).

To evaluate the welfare loss associated with local information constraints, we can simply compare the total variances of final consumption across both environments. For example, with global information, the sum of consumption variances in the above example is: $TVar_G = 3 \cdot Var\left[\frac{1}{3}\left(e_1 + e_2 + e_3\right)\right] = \sigma^2$. With local information constraints, the sum of consumption variances increases to: $TVar_L = Var\left[\frac{1}{3}e_1 + \frac{1}{2}e_2 + \frac{1}{2}e_3\right] + Var\left[\frac{1}{3}e_1 + \frac{1}{2}e_2\right] + Var\left[\frac{1}{3}e_1 + \frac{1}{2}e_3\right] = \frac{4}{3}\sigma^2$. Hence total variance increases by $\frac{1}{3}\sigma^2$ and the ratio of social welfare between global and local information corresponds to a simple function of this difference. To see this notice that, under CARA, expected (marginal) utilities are equated across all connected individuals – up to a constant that captures differences in Pareto weights. This implies that, for the simplest case in which $\lambda_i = 1$ for all $i \in N$, we can write aggregate welfare as,

$$W = -n\exp\left(-\frac{r}{n}\left(\sum_i \mathbb{E}[x_i] - \frac{1}{2}r\sum_i \mathrm{Var}(x_i)\right)\right)$$

and using the fact that, from the aggregate resource constraint $\sum_i \mathbb{E}[x_i] = 0$, we can obtain



(a) Four-Individual Transfer Network

(b) Four-Individual Information Network

Figure 2: Transfer Network Need Not Equal the Information Network

an expression for the welfare ratio between global and local constraints in terms of aggregate variances as,

$$\frac{W^G}{W^L} = \exp\left(\frac{r^2}{2n}\left(TVar_G - TVar_L\right)\right)$$

2.4 General Contractibility Constraints

The previous example has the feature that local information constraints are defined based on the underlying physical network at which transfers take place. This does not have to be the case. Our framework is more general and may be interpreted as the "reduced-form" representation of all effective contractibility constraints. It may encode, say, primitive features of the environment that may be relevant to contractibility considerations, including the extent of observability, the technology of communication, and opportunities for ex post strategic interactions that may effectively implement truthful information transmission in equilibrium. More precisely, one might imagine that messages about endowments may be distributed along pairs of linked households, such that information extends beyond the immediate neighborhoods. Alternatively, one could also consider "directed information" whereby the realization of some endowment e_k might be contractible to a pair i and j, but not vice versa.

To fix ideas, consider an augmented four-agent line network shown in panel (a) of Figure 2 and take this to be the underlying transfer network. Imagine, say, that we now allow transfers t_{12} and t_{23} to depend on e_4 as well as on direct neighbors' endowments as before. This corresponds to a situation in which information constraints are not, strictly speaking, "local", in the sense of corresponding to the endowments of connected neighbors. Nonetheless, contracts remain incomplete since transfers are not contingent on all states.

In order to represent risk-sharing contracts in this setting we construct an alternative *information network* as shown in panel (b) of Figure 2. This network is a directed super-graph of the original network, and we can now characterize transfers by considering "local information constraints"' (as defined above) of this alternative network. In other words, any pair that is connected in the original transfer network can now contract on the common information of her "in-neighbors" of the directed information network.[11] Indeed, we show in Section A.1 that we can apply our framework to information networks like these in order to extend our results to contractibility environments that do not necessarily coincide with the original transfer networks.

3 General Framework

We now turn to a general framework that extends the lessons of the previous example by characterizing the bilateral risk-sharing arrangements for any given network while allowing endowment shocks to be correlated across households.

3.1 Setup

Before we proceed to our main analysis, we first introduce the model setup and define some notations. Let $N = \{1, 2, ..., n\}$ be a finite set of individuals and let G be the adjacency matrix of a network structure on N. A pair of individuals i, j are linked if $G_{ij} = 1$, and by convention, $G_{ii} = 0$. Throughout the paper we assume, without loss of generality, that G represents a connected network.[12] Denote the neighborhood of i by $N_i := \{j \in N : G_{ij} = 1\}$ and the extended neighborhood of i by $\overline{N}_i := N_i \cup \{i\}$. Let $d_i := \#(N_i)$ denote individual i's degree. The state of the world is defined as the vector of endowment realizations $e \equiv (e_i)_{i \in N} \in \Omega \equiv \mathbb{R}^n$, and its distribution is characterized by a probability measure $\mathbb{P}$ on $(\Omega, \mathscr{B}(\Omega))$. We assume that the distribution of e has finite expectation.

We assume that only linked pairs of individuals can engage in informal risk sharing directly, and such linked pairs can *ex ante* enter into and commit to a bilateral risk-sharing arrangement.[13] An ex ante risk-sharing arrangement between linked individuals i and j is a net transfer rule $t_{ij} : \Omega \to \mathbb{R}$, which prescribes a net amount of $t_{ij}(e)$ to be transferred from i to j at each realized state e. By definition, $t_{ij}(e) = -t_{ji}(e)$ for every $e \in \Omega$ and linked $i, j \in N$.

A central feature of our model is that we impose local information constraints on the bilateral contracts each linked pair of individuals may write. For most of the paper, we adopt the specification that individuals can only observe the endowment realizations of their direct neighbors, and the bilateral contract a linked pair of individuals enter into may only be made contingent on the endowment realizations that they can commonly observe, i.e., their own and their *common* neighbors' endowment realizations. Define $N_{ij} := N_i \cap N_j$ and $\overline{N}_{ij} := \overline{N}_i \cap \overline{N}_j$.

[11] If instead the information network in panel (b) of Figure 2 was undirected, this would correspond to a different contractibilty environment in which not only would t_{12} and t_{23} depend on e_4, but t_{34} would now also depend on e_2.

[12] Otherwise we may analyze each component separately.

[13] In this paper we abstract from ex-post enforcement problems for such contracts.

Let $I_i(e) := (e_j)_{j \in \overline{N}_i}$ be the information vector of i, and $I_{ij}(e) := (e_k)_{k \in \overline{N}_{ij}}$ be the common information vector of a linked pair ij. We may later refer to I_{ij} as the local state for ij. Mathematically, the local information constraints we introduce above requires that t_{ij} varies with I_{ij} only, or that $t_{ij} : \Omega \to \mathbb{R}$ be $\sigma(I_{ij})$-measurable, where $\sigma(I_{ij})$ denotes the sub-σ-algebra induced by I_{ij}.

This specification implicitly assumes that the *information network* that encodes observability (or other forms of information transmission) of endowment realizations coincides with the *physical transfer network* that encodes the ability for two individuals to write and commit to a bilateral risk-sharing contract ex ante. However, such a restriction is non-essential for our analysis, and it is imposed here for the expositional simplicity. Section A.1 considers a more general formulation of contractibility constraints, and provide generalization of our model, as well as the results, beyond the current specification.

We refer to the profile of ex ante risk-sharing arrangements t_{ij} between all pairs of linked individuals as a transfer arrangement t. Let $\mathcal{T}$ denote the set of all admissible transfer arrangements $t : \Omega := \mathbb{R}^n \to \mathbb{R}^{\sum_{i \in N} d_i}$ that are only contingent on the local states for all linked pairs:[14]

$$\mathcal{T} := \left\{ t : \Omega \to \mathbb{R}^{\sum_{i \in N} d_i} \middle| \forall i, j \text{ s.t. } G_{ij} = 1, \quad \text{and } \begin{array}{c} t_{ij} \text{ is } \sigma(I_{ij})\text{-measurable} \\ t_{ij}(e) + t_{ji}(e) = 0, \ \forall e \in \Omega, \\ \text{and } \mathbb{E}[t_{ij}] \text{ is finite.} \end{array} \right\}$$

Define $\langle s, t \rangle := \mathbb{E}\left[\sum_{G_{ij}=1} s_{ij}(e) t_{ij}(e)\right]$ for any $s, t \in \mathcal{T}$. It follows that $\langle \cdot, \cdot \rangle$ is an inner product and $\mathcal{T}$ is a well-defined inner product space (see Lemma 1 in Appendix C.1 for a formal proof). We slightly abuse notations by treating each element in $\mathcal{T}$ as an equivalent class of transfer arrangements that are indistinguishable under the norm induced by $\langle \cdot, \cdot \rangle$. Throughout the paper, we write "$s = t$" to mean "$\langle s - t, s - t \rangle = 0$", whenever applicable.

Given a transfer arrangement $t \in \mathcal{T}$, we define the final consumption plan induced by t as $x^t : \Omega \to \mathbb{R}^n$ with $x_i^t(e) := e_i - \sum_{h \in N_i} t_{ij}(e)$. Individuals derive utilities from their own final consumption,[15] and we assume that they have a strictly concave and twice differentiable utility

[14] We clarify that transfers are not allowed to be interdependent in this paper, i.e., the net transfer between any linked pair *cannot* depend on the transfers between any other pairs. It should be pointed out that, if transfers are also observable and (informally) contractible, then essentially the final consumption becomes contractible and the full-information Pareto efficiency can be restored via state-by-state global equal sharing (potentially with some state-independent transfers), eliminating the significance of any network structure beyond connectedness and any joint endowment distribution on the final consumption plan. See, however, Appendix A.2 how our framework can be adapted to incorporate other forms of *ex post interactions*.

[15] Here we abstract away from constraints on minimum consumption levels, which clearly would further reduce the efficiency of the risk sharing contracts. However if income variances are small relative to expected income

function u, with $u' > 0$ and $u'' < 0$.

The timeline of our model is summarized as follows: *ex ante*, given a fixed network structure G, each linked pair ij enters into a bilateral risk-sharing contract t_{ij}; the endowment vector e realizes; *ex post*, each linked pair ij carries out the network transfer of amount $t_{ij}(I_{ij}(e))$ according to their ex-ante contract t_{ij} and their local information $I_{ij}(e)$; after the transfers, each individual derive utility from her final consumption $x_i^t(e)$.

The central question we seek to answer in the subsequent analysis to characterize the constrained Pareto efficient risk-sharing arrangements subject to the local information constraints.

3.2 General Conditions for Pareto Efficiency

To characterize the set of Pareto efficient transfers under the local information constraint, we consider the following problem:

$$\max_{t \in \mathcal{T}} J(t) := \mathbb{E}\left[\sum_{k \in N} \lambda_k u\left(e_k - \sum_{h \in N_k} t_{kh}\right)\right] \tag{3}$$

Recall that both e and t are assumed to have finite expectation. As u is strictly concave, by Jensen's inequality, we conclude that $\mathbb{E}\left[u\left(e_k - \sum_{h \in N_k} t_{kh}\right)\right] < \infty$ for all $k \in N$, so the social welfare function $J : \mathcal{T} \to \mathbb{R} \cup \{-\infty\}$ is well defined on $\mathcal{T}$.

The following proposition provides a formal characterization of the solution to the maximization problem above. Since the transfer rule t_{ij} is restricted to be measurable with respect to $\sigma(I_{ij})$, we slightly abuse notations and write it as $t_{ij} : \mathbb{R}^{d_{ij}+2} \to \mathbb{R}$ where $d_{ij} + 2 = \dim(I_{ij})$. We denote the distribution of I_{ij} on $\mathbb{R}^{d_{ij}+2}$ by $\mathbb{P}I_{ij}^{-1}$.

Proposition 1. *A profile of $t \in \mathcal{T}$ solves (3) if and only if it simultaneously solves the $\sum_{i \in N} d_i$ optimization problems in the form of (4) at $\mathbb{P}I_{ij}^{-1}$-almost all possible local states of the linked pair: $\forall i, j$ s.t. $G_{ij} = 1$, for $\mathbb{P}I_{ij}^{-1}$-almost all $\tilde{I}_{ij} \in \mathbb{R}^{d_{ij}+2}$,*

$$t_{ij}\left(\tilde{I}_{ij}\right) \in \arg\max_{\tilde{t}_{ij} \in \mathbb{R}} \mathbb{E}\left[\begin{array}{c} \lambda_i u_i\left(e_i - \tilde{t}_{ij} - \sum_{h \in N_i \setminus \{j\}} t_{ih}(e)\right) \\ +\lambda_j u_j\left(e_j + \tilde{t}_{ij} - \sum_{h \in N_j \setminus \{i\}} t_{jh}(e)\right) \end{array} \middle| I_{ij}(e) = \tilde{I}_{ij}\right] \tag{4}$$

Proposition 1 is an intuitive result. Its analogue under global information has a similar form and essentially connects the marginal utilities of consumption of two individuals in two different states (Wilson, 1968). With local information, the statement is now expressed, for every linked pair, in terms of a conditional expectation over the common information set of

levels, then we expect the distortions to be small.

that pair. Therefore, equation (4) says that the set of Pareto efficient transfers call for pairwise efficient risk sharing along each link of the network, where efficiency is measured with respect to an expectation over all possible realizations of the nonlocal information.

Importantly, Proposition 1 provides a motivation for investigating Pareto efficient risk sharing subject to local information by implying that these are exactly the possible outcomes resulting from decentralized negotiation procedures satisfying the weak requirement that neighboring agents end up with agreements that are efficient at the pair level. To see this, notice that Proposition 1 establishes an equivalence between Pareto-efficient risk-sharing arrangements subject to local information constraints and stable outcomes of decentralized bilateral risk sharing arrangements between neighbors subject to the same constraints. In problem (4), at each I_{ij}, the choice of t_{ij} affects the expected utilities of only i and j, so each optimization problem in (4) can be reinterpreted as the surplus maximization problem jointly solved by the linked pair ij, given the transfer rules chosen by other linked pairs. Therefore, any bargaining procedure that leads to an agreement between any two neighboring agents that is efficient for the pair (does not leave surplus on the table) given other agreements, results in a Pareto efficient outcome at the social level.[16]

The next result establishes that while in general there can be multiple transfer profiles satisfying the conditions for optimality (4), they all imply the same consumption plan in all states.

Proposition 2. *All profiles of transfers $t \in \mathcal{T}$ that solve (3) lead to ($\mathbb{P}$-almost) the same consumption plan x.*

By Proposition 2, if we can find a profile of transfers so that the induced consumption plan satisfy (4), then it must correspond to a Pareto efficient risk-sharing arrangement.

For simplicity, below we will denote the conditional expectation operator $\mathbb{E}\left[\,\cdot\,|I_{ij}\right]$ by $\mathbb{E}_{ij}\left[\,\cdot\,\right]$. Following Propositions 1 and 2, we may express the necessary and sufficient condition for Pareto efficiency as a requirement on the ratio of conditional expected marginal utilities.

Corollary 1. *A profile of transfers t is Pareto efficient if and only if the ratio of the expected marginal utilities conditional on all local states is constant: for every $i, j \in N$ s.t. $G_{ij} = 1$,*

$$\frac{\mathbb{E}_{ij}\left[u_i'\left(x_i^t\right)\right]}{\mathbb{E}_{ij}\left[u_j'\left(x_j^t\right)\right]} = \frac{\lambda_j}{\lambda_i}. \tag{5}$$

[16] A concrete example for such a negotiation procedure is split the difference negotiations, originally proposed in Stole and Zwiebel (1996) and adopted to the risk sharing context in Ambrus and Elliott (2020).

This result extends the well-known Borch rule (Borch, 1962; Wilson, 1968) for Pareto efficient risk-sharing arrangements to settings with local information constraints. As opposed to the global-information case, the ratio of expected marginal utilities need not be equal state by state and across all individuals: they only have to be equal between linked individuals and in expectation, conditional on local common information.

Before characterizing particular transfer schemes for more specific environments, we want to emphasize that Propositions 1 and 2 are derived for general convex utility functions that may be arbitrarily heterogeneous across individuals, and for general joint distribution of endowment shocks that can be arbitrarily correlated across individuals.

3.3 Efficient Risk-sharing in the CARA-Normal Setting

In this section we investigate Pareto efficient risk-sharing arrangements, subject to local information constraints, under the assumption of CARA utilities and jointly normally distributed endowments with a uniform global correlation structure.[17]

Assumption 1. *For the remainder of this section we assume that individuals have homogeneous CARA utility functions $u(x) = -\exp(-rx)$, where $r > 0$ is the coefficient of absolute risk aversion. The vector of endowments $(e_i)_{i \in N}$ follows a multivariate normal distribution, $e \sim \mathcal{N}(\mathbf{0}, \sigma^2\Sigma)$ with $\Sigma_{ii} = 1$ for all i and $\sum_{ij} = \rho$ for all $i \neq j$, for some $\rho \in \left[-\frac{1}{n-1}, 1\right]$.* [18]

To maintain analytical tractability, we assume symmetric correlation structure, where any two individuals' endowments have the same correlation coefficient $\rho \in \left[-\frac{1}{n-1}, 1\right]$. Equivalently, we are assuming that each individual's endowment can be decomposed additively into two independent components: a common shock and an idiosyncratic shock, i.e., $e_i = \sqrt{\rho}\tilde{e}_0 + \sqrt{1-\rho}\tilde{e}_i$, with $(\tilde{e}_k)_{k=0}^n \sim_{iid} \mathcal{N}(0, \sigma^2)$.

We now proceed under Assumption 1 and fully characterize the transfer rules that achieve Pareto efficient consumption profiles subject to local information constraints, for any network. We will show that these rules are linear and strictly bilateral. A linear transfer rule specifies that the transfer between any two connected individuals is a linear function of endowment realizations in the pair's joint information set. We show in the next two sections that linear transfer rules can achieve any Pareto efficient risk-sharing arrangement, where the precise linear

[17] We later consider, in Section 6.1 the implications of individual heterogeneity in endowment distribution and risk aversion within the CARA-Normal setting, as well as extensions of our results beyond the CARA-Normal setting.

[18] $-\frac{1}{n-1}$ is the lower bound for a global pairwise correlation in a n-person economy; mathematically, it is the smallest ρ such that the variance-covariance matrix is positive semi-definite. For any $\rho \in \left[-\frac{1}{n-1}, 1\right]$, the variance-covariance matrix is positive semi-definite.

form depends crucially on the level of correlation in endowments. Moreover, we show that Pareto efficient consumption allocations may be achieved by linear rules that are also *strictly bilateral* – that is, a transfer between i and j need not condition on the information of a common neighbor k. In the next two subsections we give a precise characterization for these rules as a function of the underlying network structure.

Before characterizing the sharing rules, we want to stress that while strictly bilateral transfers are not necessary to achieve efficient allocations, we do think they are the most reasonable. Optimal transfers that are not strictly bilateral circulate goods along cycles of common neighbors, with no real effect on final consumption.[19] We therefore think that these transfers are in some sense redundant and less likely than strictly bilateral transfers. Most importantly, since we show in Proposition 2 that there is a unique optimal allocation, the possibility of conditioning transfers on common neighbors is in fact irrelevant from the perspective of final consumption allocations.

3.3.1 Independent Endowments

To simplify the presentation of our results, we first analyze the case where endowments are independent, i.e., $\rho = 0$. We show that optimal transfers may be easily described as a localized version of the equal sharing rule, in which individuals transfer an equal share of their endowment to all their neighbors. We later show that adding correlation alters the formula for optimal transfers, while keeping a linear and bilateral form.

We first verify that the local equal-sharing arrangement is indeed the optimal linear rule subject to local information constraints (in $\mathcal{T}$) satisfying the expectational Borch rule. Given any linear transfer scheme, final consumption, conditional on I_{ij}, also follows normal distribution, so $\mathbb{E}_{ij}\left[u_i'\left(x_i\right)\right] = r\exp\left[-r\left(\mathbb{E}_{ij}\left[x_i\right] - \frac{1}{2}rVar_{ij}\left[x_i\right]\right)\right]$. Define the conditional certainty equivalent $CE\left(x_i\middle| I_{ij}\right) := \mathbb{E}_{ij}\left[x_i\right] - \frac{1}{2}rVar_{ij}\left[x_i\right]$. Then (5) can then be rewritten as

$$CE\left(x_i^*\middle| I_{ij}\right) - \frac{1}{r}\ln\lambda_i = CE\left(x_j^*\middle| I_{ij}\right) - \frac{1}{r}\ln\lambda_j. \tag{6}$$

The profile of transfer schemes t achieves Pareto efficiency if and only if (6) holds for every pair of ij such that $G_{ij} = 1$. Intuitively, equation (6) states that the difference in the conditional certainty equivalents is constant across all local states for a linked pair.

We say a profile of transfer rules is *strictly bilateral* if t_{ij} is $\sigma\left(e_i, e_j\right)$-measurable. We now

[19] In Appendix C.7 we show that for tree networks (i.e. cycle-free networks) the linear transfer scheme featured in Proposition 3 is the unique transfer arrangement that achieves a given Pareto efficient risk-sharing arrangement.

characterize efficient transfers subject to local information for the case of independent endowments.

Proposition 3. *Given any profile of positive welfare weights* $(\lambda_i)_{i\in N}$*, there always exists a strictly bilateral Pareto efficient profile of transfer rules in* $\mathcal{T}$ *in the form of the following "local equal sharing rules":*

$$t_{ij}^{*}(e_i, e_j) := \frac{e_i}{d_i+1} - \frac{e_j}{d_j+1} + \mu_{ij}^{*},$$

for some constant $\mu_{ij}^{*} \in \mathbb{R}$*, for each linked pair* ij*.*

Proposition 3 shows that the efficient transfer $t_{ij}^{*}(e_i, e_j)$ subject to the local information constraint is composed of two parts: the state-contingent "sharing rule" and the state-independent "insurance premium" (captured by $\mu_{ij}^{\star}$), just like in the simple example above. The state-contingent transfer scheme corresponds to the *local equal sharing rule* in which i transfers a fraction $1/(d_i+1)$ of her endowment to each of her friends and receives a fraction $1/(d_j+1)$ from each friend j.

Notice that this transfer scheme is linear in endowments and that only bilateral information is required for efficient risk sharing with local information. Also, this proposition suggests that two linked individuals ij only require ex ante knowledge of the local network structure (more precisely d_i and d_j) to compute and contract on the socially optimal transfer rule t_{ij}^{*}.

3.3.2 Correlated Endowments

Formally, notice that the joint information of individuals i and j affects the conditional distribution of some non-common endowment k as follows

$$e_k|_{e_i,e_j} \sim \mathcal{N}\left(\frac{\rho}{1+\rho}(e_i+e_j),\ \frac{1+\rho-2\rho^2}{1+\rho}\cdot\sigma^2\right) \tag{7}$$

Setting $\rho = 0$ implies that $e_k|_{e_i,e_j} \sim \mathcal{N}(0,\sigma^2)$ as in the previous section, leading to local equal-sharing as the optimal transfer rule. In this section we show exactly how the local equal-sharing rule is affected by the presence of correlated endowments. We show that linear and strictly bilateral rules of the form

$$t_{ij}(e_i, e_j) = \alpha_{ij}e_i - \alpha_{ji}e_j + \mu_{ij} \quad \text{for all } ij : G_{ij} = 1 \tag{8}$$

still achieve Pareto efficient allocations – as in the previous section – and we provide precise characterization for the coefficients α_{ij} as a function of the network and the correlation parameter ρ.

For clarity, we first illustrate the main ideas for the case of *minimally-connected networks*. Notice that, under minimal connectedness, $I_{ij} = (e_i, e_j)$, so transfer t_{ij} must be strictly bilateral. Then, the local FOC for optimality in equation (5) can be written as

$$\begin{aligned} t_{ij} = \frac{1}{2}e_i - \frac{1}{2}e_j - \frac{1}{2r}\ln\mathbb{E}\left[\exp\left(r\sum_{k\in N_i\setminus\{j\}} t_{ik}\left(e_i,e_k\right)\right)\middle| e_i, e_j\right] \\ + \frac{1}{2r}\ln\mathbb{E}\left[\exp\left(r\sum_{k\in N_j\setminus\{i\}} t_{jk}\left(e_j,e_k\right)\right)\middle| e_i, e_j\right] + \frac{1}{2r}\ln\frac{\lambda_j}{\lambda_i} \end{aligned} \tag{9}$$

Postulating a linear transfer scheme of the form, $t_{ij}\left(e_i, e_j\right) = \alpha_{ij}e_i - \alpha_{ji}e_j + \mu_{ij}$, $\forall G_{ij} = 1$, we can substitute the postulated linear forms of t_{ik} into (9) and obtain expressions for the above conditional expectations in terms of linear combinations of endowments based on the conditional distribution given in (7). We can therefore explicitly derive the conditional expectation terms in the above formula and, after collecting terms and reconciling with the postulated formula for t_{ij}, arrive at the following system of equations:[20]

$$\alpha_{ij} = \frac{1}{2}\left[1 - \sum_{k\in N_i\setminus\{j\}}\alpha_{ik} + \frac{\rho}{1+\rho}\left(\sum_{k\in N_i\setminus\{j\}}\alpha_{ki} - \sum_{k\in N_j\setminus\{i\}}\alpha_{kj}\right)\right] \quad \forall ij \text{ s.t. } G_{ij} = 1. \tag{10}$$

In equation (10), the net transferred share α_{ij} of e_i from i to j is given by the half of the "remaining share" after deducting the transfers to i's other neighbors $N_i\setminus\{j\}$, corrected by an adjustment for inflows of non-local endowments. The $\frac{1}{2}$ multiplier is analogous to the equal sharing rule in the independent endowments case, but last term in the square brackets is new (i.e. it disappears when $\rho = 0$). We refer to this term as an *informational effect*, for the following reason: $\sum_{k\in N_i\setminus\{j\}}\alpha_{ki}$ is the sum of i's shares of i's other neighbors' endowments $\left(e_k\right)_{k\in N_i\setminus\{j\}}$, and the conditional expectation of each k's endowment changes linearly with the realization of e_i by a factor of $\frac{\rho}{1+\rho}$. Similarly, $\sum_{k\in N_j\setminus\{i\}}\alpha_{kj}$ is the sum of j's shares of j's other neighbors' endowments $\left(e_k\right)_{k\in N_j\setminus\{i\}}$, and the conditional expectation of each k's endowment also changes linearly with the realization of e_i by a factor of $\frac{\rho}{1+\rho}$. Intuitively, due to the symmetric correlation structure, the realization of e_i provides the same amount of local information about

[20]Rigorously there should be another set of equations that verify the guess for the state-independent constant transfers μ, which in general involve both α and μ. However, Lemma 6 in Appendix B.4 implies that, given any admissible α, there exist some μ such that (α, μ) satisfies the set of verification equations for the constant transfers. Hence, system (10) (which involves only α) constitutes the essential condition for Pareto efficiency. We therefore omit the conditions on μ and delay our discussion about state-independent transfers to Section A.3.

all non-local endowments e_k for $k \notin \overline{N}_{ij}$, and thus its informational effect can be calculated as a simple net sum of endowment shares. Finally, since a larger e_i predicts that both i and j are more likely to obtain higher amounts of inflows from uncommon neighbors, this commonly recognized information can be used by the pair ij to (imperfectly) share the non-local risk exposures.[21] After pooling the conditional expectations of non-local inflows, i and j again share the *remaining* shares of e_i and e_j equally.

Notice that every individual i carries out this kind of "equal sharing" behavior with all her neighbors, and the inflow/outflow shares (α_{ij}) must make all the sharing simultaneously equal (in expectation). In other words, solving for the transfer in (8) explicitly involves solving conditions (10) simultaneously to obtain the full profile of bilateral shares, (α_{ij}). We do this below for *general network structures*, which obviously include the case of minimally connected networks. However, for general network structures transfers need not be strictly bilateral. A general analysis must allow for this possibility.

In Appendix B.5, we show that, for general network structures, a linear and strictly bilateral profile of transfer rules that solves a system of linear equations that encodes the local Borch rule is indeed Pareto efficient. However, the required system of equations proves difficult to solve directly. Instead, we show via Lemma 8 that we may *equivalently solve* an alternative optimization problem that minimizes total consumption variances among *linear* transfer rules only – this problem is defined formally below in equation (11).

Although the equivalence result in Lemma 8 is involved, we can provide an intuitive explanation for why the planner problem may be equivalently formulated as an aggregate variance minimization problem – as long as there exists a *linear* profile of transfer rules that is Pareto efficient.[22] Specifically, we make use of the fact that, under CARA utility, equation (6) shows how the difference in certainty equivalent consumption is constant across (local) states. We can therefore rewrite the planner problem as the maximization of a representative agent's utility. To fix ideas, take the case where $\lambda_i = \lambda_j$ for all $i, j \in N$. Then, for all $i \in N$, $CE_i = \overline{CE} := \frac{1}{n}\sum_i CE_i$. With this, we can rewrite the welfare maximization problem of equation (3) in terms of $\overline{CE}$ only,

$$J(t) = n \cdot u(\overline{CE})$$

Now, if we restrict ourselves to *linear* transfer rules of the form of (8), then final consumption

[21] To be precise, by "inflow" we mean the undertaking of a share of someone else's income endowment, which may be positive or negative; by "outflow" we mean the distribution of a share of one's own endowment to someone else, which may also be positive or negative. In particular, a negative inflow is not the same as an outflow. Instead, i's inflow from j is the same as j's outflow to i.

[22] The technical details of Lemma 8 are required to demonstrate that there indeed exists a linear profile of transfer rules that solves conditions (10) and is therefore Pareto efficient.

is known to be normally distributed and $\overline{CE} = \frac{1}{n}\sum_i \left(\mathbb{E}[x_i] - \frac{1}{2} r \operatorname{Var}(x_i)\right)$. By the aggregate resource constraint, $\sum_i \mathbb{E}[x_i] = 0$, which means we can rewrite the above problem as,

$$\max_{t\in\mathcal{T}} J(t) = -\exp\left(\frac{r^2}{2n}\sum_i \operatorname{Var}(x_i)\right)$$

and this corresponds to the minimization of aggregate consumption variance.[23]

We now proceed to solve the variance minimization problem. Specifically, let α be a linear profile of transfer rules in $\mathcal{T}$. The equivalent problem to solve is now given by,

$$\min_{\alpha} \sum_{i\in N} Var\left[\alpha_{ii} e_i + \sum_{jN_i} \alpha_{ji} e_j\right]. \tag{11}$$

with $\alpha_{ii} := 1 - \sum_{j\in N_i} \alpha_{ij}$ represents individual i's exposure to i's own endowment shock.

Let $\tilde{\Lambda}_i$ be the Lagrange multiplier associated with i's outflow constraint $\sum_{j\in\overline{N}_i} \alpha_{ij} = 1$ and denote $\Lambda_i := \frac{\tilde{\Lambda}_i}{2(1-\rho)}$. Then, taking the FOC for the Lagrangian w.r.t. $(\alpha_{ij})_{i\in N, j\in\overline{N}_i}$, we have

$$\begin{cases} \alpha_{ji} = \Lambda_j - \frac{\rho}{1-\rho}\left(\alpha_{ii} + \sum_{k\in N_i} \alpha_{ki}\right) \quad \forall j \in \overline{N}_i, \forall i \in N & (12.1) \\ \sum_{j\in\overline{N}_i} \alpha_{ij} = 1 \quad \forall i \in N & (12.2) \end{cases} \tag{12}$$

This is a system of $(\sum_i d_i + 2n)$ equations in $(\sum_i d_i + 2n)$ variables (α, Λ).

With all this, we are now equipped to characterize the set of Pareto efficient linear and bilateral transfer rules by obtaining the complete profile of bilateral shares (α_{ij}) that solve the above problem, and substituting them into equation (8). We present the following main characterization result:

Proposition 4. *For any $\rho \in \left(-\frac{1}{n-1}, 1\right)$ and any network structure G there exists a unique solution to system (12) given by the following: $\forall i \in N$, $\forall j \in \overline{N}_i$,*

$$\alpha_{ji} = \Lambda_j - \frac{\rho}{1+\rho d_i}\sum_{k\in\overline{N}_i} \Lambda_k \tag{13}$$

[23] Under a general distribution of Pareto weights, $(\lambda_i)_{i=1}^n$, expected transfers will vary in order to satisfy equation (6), but the contingent shares of the linear transfer rule (i.e. the α's) will still minimize aggregate variance. To see this notice that with heterogeneous λ's, $CE_j = \overline{CE} + \frac{1}{nr}\sum_i (\ln\lambda_j - \ln\lambda_i)$ for all $j \in N$, and therefore

$$J(t) = \sum_i \lambda_i u\left(\overline{CE} + \frac{1}{nr}\sum_k (\ln\lambda_i - \ln\lambda_k)\right) = \sum_i \lambda_i u\left(\overline{CE}\right) u\left(\frac{1}{nr}\sum_k (\ln\lambda_i - \ln\lambda_k)\right) = u\left(\overline{CE}\right) f(\lambda_1, \ldots, \lambda_n)$$

where $f(\lambda_1 \ldots, \lambda_n)$ is not affected by the choice of α.

where Λ_i is defined by any of the following equivalent representations:

- *(Fixed point representation):*

$$\Lambda_i = \frac{1}{d_i+1}\left(1 + \sum_{j\in\overline{N}_i}\sum_{k\in\overline{N}_j}\frac{\rho}{1+\rho d_j}\Lambda_k\right) \tag{14}$$

- *(Closed-form representation): writing $\Lambda = (\Lambda_i)_{i=1}^n$*

$$\Lambda = \left(\overline{D} - \overline{G}\Psi\overline{G}\right)^{-1}\mathbf{1}$$

where $\overline{D}$ is a diagonal matrix with its i-th diagonal entry being d_i+1, Ψ is a diagonal matrix with its i-th diagonal entry being $\frac{\rho}{1+\rho d_i}$, and $\overline{G} := G + \mathbf{I}_n$.

- *(Explicit representation): For $\rho \in [0,1)$,*

$$\Lambda_i = \frac{1}{d_i+1} + \sum_{q\in\mathbb{N}}\sum_{j\in N}\sum_{\pi_{ij}\in\Pi_{ij}^{2q}} W(\pi_{ij}) \tag{15}$$

where $W(\pi_{ij})$, the weight of each path $\pi_{ij} = (i_0, i_1, i_2, \ldots i_q)$ of length q from i to j (i.e. $i_0 = i$ and $i_q = j$), is given by,

$$W(\pi_{ij}) := \frac{1}{d_{i_0}+1}\cdot\frac{\rho}{1+\rho d_{i_1}}\cdot\frac{1}{d_{i_2}+1}\cdot\frac{\rho}{1+\rho d_{i_3}}\cdot\ldots\cdot\frac{1}{d_{i_q}+1} \tag{16}$$

The above result provides the first closed-form prediction of risk-sharing transfers on a general network of informal insurance that we know of in the literature.[24] This result includes the above case of independent endowments, but also allow for more sophisticated transfer rules that account for uniform correlations. Indeed, notice that for $\rho = 0$ equation (13) specifies that $\alpha_{ji} = \Lambda_j$ for all $i \in \bar{N}_j$, which necessarily implies that $\alpha_{ji} = 1/(d_j+1)$, as stated in Proposition 3. The way in which the presence of correlated endowments affects the shape of bilateral shares α_{ij} depends on the network structure in complicated ways, as captured by Λ_i. This *centrality measure* summarizes individuals' relevant network position, by aggregating indirect effects that are interconnected across neighbors.

To obtain intuition for the "network features" contained in Λ_i, notice that the fixed point representation in equation (14) expresses each individual's centrality recursively as a function

[24] See Appendix C.8 for Pareto efficient risk-sharing arrangements in the boundary cases of $\rho \in \left\{-\frac{1}{n-1}, 1\right\}$.

of the centrality of their neighbors' neighbors. This suggests that interactions at distance two (i.e. neighbors of neighbors) are directly relevant in this setting. To see this notice that the the network interaction terms in (12.1) define the shares going in to j as substitutes of one another. This implies that individuals *at most two links apart* (i.e. with a common neighbor, j) respond negatively to each others' shares directly through this optimal trade-off.

Moreover, indirect effects play a crucial role here as well. To see this, notice that two households with a common neighbor j not only interact through their transfer to j, but might also exchange resources with other partners, and these other relations affect what j receives from them, given their constraints in (12.2). This is the main message behind equation (13), where these inter-dependencies along the network have been solved for, and we can express the share from i to j as a function of some constants Λ's, that accumulate all these indirect effects.

The recursive representation in (14), is reminiscent of Katz-Bonacich, Page Rank, and other global network measures, albeit with two crucial differences: 1) The centrality of i depends on the centralities not of direct neighbors, but of neighbors of neighbors, and 2) the weights are not a simple geometric series (as in the Bonacich measure), but instead depend explicitly on the degree of the direct neighbors that are linking i with all of her length-two neighbors.

Solving for Λ_i in (14) provides an alternative representation of the centrality as the accumulation of *weighted even paths*, shown in (15). This expression also reflects the two main differences with standard measures (i.e. length two and path-specific weights). To see this notice first that the explicit representation of (15) defines i's centrality as the accumulation of weighted paths *of even length* starting from i. Second, notice that the weights given in (16) account for the degree of all individuals involved in a given path.

However, notice that (14) sums over individuals in $\bar{N}_i$ and $\bar{N}_j$. In other words, self-loops are allowed. This implies that we are not in a situation where an individual that is, say, at distance 3 from i will not matter for i's centrality measure. On the contrary, she will in fact matter because self-loops will allow us to reach any individual that is weakly connected to i. The weighting scheme, however, will depend crucially on the even-length paths that we can compute, starting from i. In other words, while this measure ultimately relates individuals at all distances in the network, the specific weights between each pair of individuals require counting only the even-length paths that connect them.

This complicated weighting scheme unfortunately makes comparative statics on the network structure difficult to analyze. To see this notice that when a link is removed from the network, a number of even-length paths disappear, lowering the total elements being summed in (15). However, this also lowers the degree of the two individuals involved in that link. This increases the weights associated to all even-length paths that go through either of these two individuals, as shown in equation (16). It is therefore difficult to know, in general, the way in which the centrality measure responds to changes in the network structure.

Lastly, we briefly discuss how social welfare, as measured by the total variance of final consumption across all individuals, varies with the correlation parameter.[25] On the one hand, as the correlation parameter ρ becomes more positive, the scope for risk reduction via risk sharing gets smaller. At the extreme of perfect correlation, only uninsurable aggregate risk remains. On the other hand, as $|\rho|$ increases, local information becomes more informative about nonlocal endowment realizations, reducing the loss of surplus caused by the local information constraints. The combination of the two effects make it challenging to explicitly analyze the highly nonlinear way in which total variance varies with the correlation parameter ρ in general. In Appendix C.9, we show that, in star networks, the total variance is an increasing but concave function of ρ. This illustrates that the first effect described above dominates, resulting in the overall increasingness, while the second effect is also present, leading to the concavity.[26]

4 Network Centrality and Consumption Volatility

In this section, we present an important implication of our model concerning the relationship between network centrality and consumption volatility in risk-sharing communities. Indeed, a crucial advantage of obtaining closed-form predictions on transfers is that we can provide clean and precise characterizations of the role of network heterogeneity on certain important features of ex-post consumption. We focus on consumption volatility, as this is an easily observable measure that highlights how network heterogeneity translates to differences in individual risk exposures under the local information constraints.

We show that, as suggested in the simple three-individual example above, more central individuals function as insurance providers to more peripheral individuals. As a result, they absorb larger shares of endowment risk than they can unload on others, leading to more overall consumption volatility, for which they may get compensated via state independent transfers.

We first derive analytical results for the correlation between network centrality and consumption volatility according to our theoretical model. For star networks, we derive closed-form formula for individual consumption variances under any endowment correlation parameter, and

[25]In addition, one may consider the comparative static of social welfare with respect to the risk-aversion parameter r, which is, however, trivial: fixing any profile of transfer shares (and consequently any final consumption plan), when r gets larger, every individual's expected utility decreases due to a greater level of absolute risk aversion, and hence the social welfare decreases.

[26]In Appendix C.9, we also show that, under any network structure, the total variance must be locally increasing on a neighborhood around $\rho = 0$.

show that the center's final consumption is always more volatile than the peripherals'. Alternatively, assuming that endowments are independent and that networks are sampled according to an Erdős-Rényi random graph generating process, we derive exact formula for the asymptotic covariance between degree centrality and consumption variance, which turns out unambiguously positive.

4.1 Star Networks

We first provide analytical results for the positive relationship between network centrality and consumption volatility in star networks.

Let c denote the center individual, who is connected to $n-1$ peripheral individuals, and none of the peripheral individuals are connected to each other. We use p to refer to a generic peripheral individual.

It is straightforward to show that a linear risk-sharing arrangement achieving Pareto efficiency subject to local information constraints specifies the following endowment shares to be transferred:

$$\alpha_{cp} = \frac{2+2(n-1)\rho}{n(2+n\rho)}, \quad \alpha_{pc} = \frac{1+\rho}{2+n\rho}, \quad \gamma_{cp} = \frac{(n-2)\rho}{2+n\rho}.$$

It follows that the difference in consumption variances in efficient contracts satisfies

$$Var(x_c) - Var(x_p) = \frac{(n-2)(1+(n-1)\rho)(1-\rho^2)}{(2+n\rho)^2} \geq 0$$

with equality only at $\rho \in \left\{-\frac{1}{n-1}, 1\right\}$. In particular, $Var(x_c) - Var(x_p) \to \frac{1-\rho^2}{\rho}$ as $n \to \infty$, and hence the consumption variance of the center can be much higher than the consumption variance of a periphery individual when ρ is low and n is high.

4.2 Erdős-Rényi Random Graphs

We now proceed to characterize the (large-network) asymptotic relationship between network centrality and consumption volatility under the Erdős-Rényi random graph setting, which lends great tractability to the analysis.

To formalize our results, write $\mathbb{P}^{ER}, \mathbb{E}^{ER}$ as the probability measure and expectation operator with respect to the Erdős-Rényi random graph generating process $\mathcal{G}^{ER}(n,p)$: for each $n \geq 2$ and $p \in (0,1)$, let

$$G_{ij} \equiv G_{ji} \sim_{i.i.d.} \; Bernoulli(p), \; \forall i,j \in \{1,...,n\}.$$

Fixing a sequence of $\{p_n\} \subseteq (0,1)$, we write $\mathbb{P}_n^{ER}, \mathbb{E}_n^{ER}$ for the Erdős-Rényi random graph generating process $\mathcal{G}^{ER}(n, p_n)$.

For each network structure G_n drawn from $\mathbb{P}_n^{ER}$, we write $d_i(G_n)$ as individual i's degree in G_n. We write e to denote a generic realization of the endowment vector, and take the distributions of e and G_n to be statistically independent from each other. Furthermore, we focus on the simple case with independent endowment shocks, i.e., the global correlation parameter $\rho = 0$. By previous results, we know that any Pareto efficient risk-sharing arrangements take the form of the local equal sharing rule, so that the final consumption allocation is given by

$$x_i(G_n)[e] := \sum_{j \in \overline{N}_i(G_n)} \frac{1}{d_j(G_n) + 1} e_j,$$

and the individual consumption variance is given by

$$Var(x_i(G_n)) = \sum_{j \in \overline{N}_i(G_n)} \frac{1}{(d_j(G_n) + 1)^2}$$

where $Var(\cdot)$ denotes the variance operator with respect to the endowment shocks e conditional on realized network structure being G_n.

Proposition 5. *Let $\{G_n\}$ be a sequence of Erdős-Rényi random graphs generated by $\mathcal{G}^{ER}(n, p_n)$.*

- *(Dense Case) Suppose $p_n = p$ for all n. Then:*

$$\lim_{n \to \infty} nCov_n^{ER}[Var(x_i(G_n)),\ d_i(G_n)] \to \frac{1-p}{p} > 0.$$

- *(Sparse Case) Suppose $np_n \to \lambda > 1$. Then:*

$$\lim_{n \to \infty} Cov_n^{ER}[Var(x_i(G_n)),\ d_i(G_n)] = \kappa(\lambda) := \mathbb{E}\left[\frac{\xi^3 + 4\xi^2 + 2(2-\lambda)\xi - 3\lambda}{(\xi+1)^2(\xi+2)^2}\right],$$

where $\xi \sim Poisson(\lambda)$. Numerical computation of $\kappa(\lambda)$ shows that $\kappa(\lambda)$ is positive whenever λ exceeds a threshold $\overline{\lambda} \approx 3.8803$.

To see the intuition behind Proposition 5, fix an individual i with d_i neighbors in some network G, and consider adding a link between i and some individual j, who has d_j neighbors in G but is originally not a neighbor of i in G (that is, consider the network $G+ij$). Notice that adding the new neighbor j for risk sharing has two opposite effects on individual i's consumption variance $Var(x_i)$. On one hand, there is one more individual, namely j, to share individual i's income shock, reducing i's exposure to her own income shock from $\frac{1}{d_i+1}e_i$ to $\frac{1}{d_i+2}e_i$ under the local equal sharing rule. On the other hand, individual i is now exposed to a share of j's income shock, $\frac{1}{d_j+2}e_j$, which individual i has zero exposure to in the original network G. Hence, the net effect of the additional link ij on individual i's consumption variance is:

$$\begin{aligned} Var\left(x_i\left(G+ij\right)\right) - Var\left(x_i\left(G\right)\right) &= \frac{1}{\left(d_j+2\right)^2} - \left[\frac{1}{\left(d_i+1\right)^2} - \frac{1}{\left(d_i+2\right)^2}\right] \\ &= \frac{1}{\left(d_j+2\right)^2} - \frac{2d_i+3}{\left(d_i+1\right)^2\left(d_i+2\right)^2} = O\left(d_j^{-2}\right) - O\left(d_i^{-3}\right). \quad (17) \end{aligned}$$

In Erdős-Renyi random graphs, d_i and d_j are both stochastically of the order of $O_p\left(np_n\right)$, so the addition in variance induced by the link ij, is stochastically of a larger order of magnitude $O_p\left(\left(np_n\right)^{-2}\right)$ than the reduction in variance $O_p\left(\left(np_n\right)^{-3}\right)$.

In the dense case where $p_n = p$, the first effect dominates in the limit: *on average*, having an additional friend for risk sharing increases one's own consumption variance in large Erdős-Rényi random graph, so that larger degree centrality is asymptotically positively correlated with consumption variance.

The sparse case, however, might be more theoretically informative and empirically relevant. In this case, where $np_n \to \lambda > 1$, the limit degree distribution is characterized by a Poisson distribution parameterized by the *limit average degree* λ. Numerical calculation indicates that, if the limit average degree λ is large enough (at least $\overline{\lambda} \approx 3.88$), then the imbalance in the orders of magnitudes between the two opposite effects is sufficiently pronounced, at which point the asymptotic correlation between consumption variance and degree centrality becomes positive. Given that average degrees in many real-world networks are conceivably larger than $\overline{\lambda} \approx 3.88$, our theory predicts a positive asymptotic correlation between degree centrality and consumption volatility for most real-world networks even under the sparse (Poisson) asymptotics. As the sparse-case result provides a sharp quantification of the threshold on *network density* at which the asymptotic correlation between degree centrality and consumption variane becomes positive, it follows that all "moderately sparse" Erdős-Rényi random graph models under which $np_n \to \infty$, i.e., whenever p_n converges to zero at a rate slower than n^{-1}, the asymptotic correlation should remain positive.

Proposition 5 suggests that, under local information constraints, more central individuals tend to undertake larger consumption variances, effectively playing the roles of "quai-insurance providers". Even though Proposition 5 is derived in the setting of Erdős-Rényi random graphs, which lends great analytical tractability, the key economic driver of the result, that is, the difference in the orders of magnitudes between the two opposite effects of an additional neighbor on an individual's consumption variance as captured by equation (17), clearly remains present beyond the setting of Erdős-Rényi random graphs. Heuristically, we expect that the asymptotic correlation remains to be positive in a large class of "sufficiently dense" network formation models, including the graphon model and most versions of the stochastic block models.

The analytical result we derived here is based on the *asymptotic* distribution of *Erdős-Rényi random graphs.* To investigate whether such large-sample results remain relevant in *finite real-world* network structures, we run simulations of our model using two real-world village networks in India from two different data sets, each randomly selected and provided to us by the researchers who collected the data.[27]

We computed the sample correlations between degree/eigenvector centrality and consumption variance, found that the correlations are positive and statistically significant in both simulations. For the data set provided by Field and Pande, the sample correlation is 0.1994, while for the data set provided by Banerjee, Chandrasekhar, Duflo and Jackson, the correlation is 0.2084. Both results are highly statistically significant with p-values at orders of magnitudes below 10^{-10}.[28]

Proposition 5 provides a sharp theoretical prediction of the model that is not only empirically feasible but also computationally easy to test. Such practicality concerns are highly relevant in the network literature, as it is often challenging to conduct direct empirical tests of micro-founded theoretical results, especially when the result involves the whole network structure or complicated network statistics. However, Proposition 5 provides a direct prediction on a simple relationship (positive correlation) between consumption volatility and one of the simplest forms of network statistics, degree centrality. In a follow-up paper – currently work in progress– we conduct a thorough empirical investigation of Proposition 5 and other potential drivers for the positive correlation between degree centrality and consumption volatility, using various different data sets from numerous village economies.

[27]The first network was provided to us by Erica Field and Rohini Pande, who collected it from villages in the districts of Thanjavur, Thiruvarur and Pudukkotai (Tamil Nadu) in India. In a subset of the villages, complete within-village network data was collected by surveying all households. The second network is from data collected by Abhijit Banerjee, Arun Chandrasekhar, Esther Duflo and Matthew Jackson in Karnataka, India (they collected complete within-village network data in 75 villages), used for example in the Banerjee, Chandrasekhar, Duflo, and Jackson (2018). From both datasets we received the network of financial connection for one randomly selected village with complete network data. From the original network we created the undirected "AND" network, that is, we defined a link between two households whenever both households indicated each other as a borrowing relationship. We excluded households that became isolated in the "AND" network.

[28]In both simulations, we randomly drew the endowment $e_i^{(t)}$ of each household according to the standard normal distribution for $T = 5000$ times: $\left\{e_i^{(t)}\right\}_{i,t} \sim_{iid} \mathcal{N}(0,1)$. We assumed that all households have CARA utility functions with $\lambda = 1$. We then computed the final consumption of each household under the equally-weighted Utilitarian optimal risk-sharing arrangement subject to local information constraints, using the results from subsection 4.1, and the sample variance of final consumption for each household (note that the variance does not depend on the planner's weights).

5 Implications of the Theory for Empirical Tests of Risk Sharing

The performance of risk-sharing communities has been repeatedly tested in data since the work of Cochrane (1991), Mace (1991) and Townsend (1994). Their original approach developed empirical tests of full insurance that related household consumption and income. Indeed, the well known Borch rule – equating the ratio of marginal utilities across households – imposes that, under full insurance, household consumption should not respond to idiosyncratic movements in income after controlling for aggregate shocks. This implication can be tested in the following popular regression:[29]

$$c_{it} = \alpha_i + \beta_1 y_{it} + \beta_2 \bar{y}_t + \epsilon_{it} \tag{18}$$

where c_{it} and y_{it} correspond to household $i's$ consumption and income at time t, and where $\bar{y}_t = \sum_i y_{it}$ represents aggregate village income at time t. Full insurance in this specification implies that $\beta_1 = 0$. An overwhelming proportion of studies have rejected the full-insurance hypothesis in a wide number of settings. As a result, a great deal of work has followed, that seeks to explain this stylized fact.

On the theory side, we have argued that this paper complements an ongoing effort to model the relevant contracting frictions in informal risk sharing environments.[30] In this section we argue that our framework also responds to a recent strand of the literature that suggests modifying the classical Townsend test in order to accommodate various forms of heterogeneity. Some of this work argues that the standard consumption regression in (18) is misspecified if, for instance, households hold heterogeneous risk preferences.[31] More relevant to the current discussion, several other studies have also suggested that households within a village indeed access different risk sharing groups, and that controlling for aggregate-level shocks, as in (18), would incorrectly estimate income coefficients: $\bar{y}_t$ should instead be a local aggregate specific

[29] The specification in equation (18) is just one of several equivalent ways to empirically test for efficient insurance. Mace (1991) obtains an equivalent formulation of (18) based on aggregate consumption by summing the Borch-rule over all households and obtains: $c_{it} = \bar{c}_t + \frac{1}{\gamma}\left(\log(\lambda_i) - \bar{\lambda}\right)$. We are essentially running this specification but we replace aggregate consumption by aggregate income, which is equivalent under the assumption of no savings. Taking first differences allows one to drop the household-specific fixed effects to obtain $\Delta c_{it} = \Delta \bar{c}_t$ which motivates another well-known test of full insurance. However, most studies avoid using village consumption as a regressor since it may mechanically equate to unity if the sample is large enough (see Shrinivas and Fafchamps (2018)). Deaton (1992) and Ravallion and Chaudhuri (1997) use village or time fixed effects (as we do in equation (18)), Townsend (1994) uses the deviation of household consumption from the village average, and Suri (2005) uses a contrast estimator adapted from the peer effects literature.

[30] For example Thomas and Worrall (1990), Kocherlakota (1996), Ambrus, Mobius, and Szeidl (2014), and Kinnan (2017).

[31] See for instance Mazzocco and Saini (2012) and Schulhofer-Wohl (2011).

to each household. In a couple well-known examples, Mazzocco and Saini (2012) argue that the relevant sharing group in India is the caste (rather than the village), while Attanasio, Meghir, and Mommaerts (2018) test for efficient insurance within extended families in the U.S.[32]

This paper refines and generalizes the modified tests that evaluate the performance of insurance mechanisms on local sharing groups. Rather than taking groups as separate, perfectly insured communities, the current framework allows for a fully general social structure with interconnected sharing groups that are specific to each household, and which may overlap in complicated ways along any given network. We show how, under the local information constraints of our model, not defining the relevant local sharing group biases the estimates of risk-sharing tests. More importantly, we show that controlling for this bias will not eliminate the correlation between household consumption and income: the structure of the network, coupled with the information constraints, induces imperfect risk-sharing and generates heterogeneity in sharing behavior. The current framework therefore allows us to decompose the standard Townsend coefficient β_1 into an underlying distribution of household-specific coefficients that capture the varying risk-sharing possibilities induced by the network structure, and which can be interpreted economically in terms of consumption volatility (as shown in the previous section).

To fix ideas, consider the simple network with three individuals and independent endowments from section 2 and set $\lambda_i = \lambda_j$ for all $i, j \in N$; all arguments below can be extended to general networks, correlated endowments, and any profile of Pareto weights. If we write down final consumption for each household in the form of the classical risk-sharing specification of equation (18), we have that,

$$\begin{cases} c_{1t} = \alpha_1 + \left(\frac{1}{3} - \frac{1}{2}\right) y_{1t} + \frac{1}{2}\bar{y}_t + \epsilon_{1t}, \\ c_{2t} = \alpha_2 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{2t} + \frac{1}{3}\bar{y}_t + \left(\epsilon_{2t} - \frac{1}{3}y_{3t}\right), \\ c_{3t} = \alpha_3 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{3t} + \frac{1}{3}\bar{y}_t + \left(\epsilon_{3t} - \frac{1}{3}y_{2t}\right), \end{cases}$$

where $\alpha_1 = \frac{1}{12}r\sigma^2$ and $\alpha_2 = \alpha_3 = -\frac{1}{24}r\sigma^2$ correspond to state-independent transfers and are represented as household-specific intercepts. These equations reflect three important themes of this paper as they relate to empirical tests of risk-sharing: 1) coefficients on own income are generically different from zero for all households. i.e. $\alpha_{ii} \neq \alpha_{ij}$, 2) these coefficients vary according to households' network position, and 3) imposing the common sharing group on all

[32]In similar procedures Hayashi, Altonji, and Kotlikoff (1996) consider whether extended families can be viewed as collective units sharing risk efficiently. Munshi and Rosenzweig (2016) also find that the caste is the relevant group to explain migration patterns in rural India. Most relevant here, Fafchamps and Lund (2003) address the failure of efficient insurance in the data suggesting that households receive transfers not at the village level, but from a network of family and friends

households generates biased estimates: notice the last two equations contain weighted incomes in the error term. The classical risk sharing test in (18) pools these equations and obtains a unique estimate for β_1; given the previous discussion we expect this estimate to be biased, different from zero, and positive.

Consider estimating (18) with the relevant local sharing group instead. In this case, we show that we still obtain heterogeneous estimates, β_i, for the coefficients on own income. As a result, running a pooled regression that estimates a unique coefficient of β_1 imposes a restriction that biases the estimator, which means that controlling for local averages is not enough to obtain a consistent estimator of β_1 when local sharing groups overlap on a network – it is important to account for household-specific coefficients on income. We argue that this may have profound implications for the type of conclusions one draws from data. To see this, rewrite again our consumption equations in the form of (18), but now allow for household-specific aggregates, $\bar{y}_{it} = \sum_{j \in N_i} y_{jt}$, that sum over the incomes of $i's$ sharing partners. In this case we have,

$$\begin{cases} c_{1t} = \alpha_1 + \left(\frac{1}{3} - \frac{1}{2}\right) y_{1t} + \frac{1}{2}\bar{y}_{1t} + \epsilon_{1t}, \\ c_{2t} = \alpha_2 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{2t} + \frac{1}{3}\bar{y}_{2t} + \epsilon_{2t}, \\ c_{3t} = \alpha_3 + \left(\frac{1}{2} - \frac{1}{3}\right) y_{3t} + \frac{1}{3}\bar{y}_{3t} + \epsilon_{3t}. \end{cases}$$

Because aggregate income terms are now household-specific (i.e. $\bar{y}_i$), the additional terms in the error disappear and we obtain unbiased estimators for individual coefficients on own income, β_i. Notice, however, that coefficients to own income are heterogeneous and different from zero as long as $\alpha_{ii} \neq \alpha_{ij}$ – some, in fact, are actually negative. The pooled coefficient, β_1, represents the average of the underlying heterogeneity in risk-sharing possibilities across households. In this example, a pooled regression will deliver a positive coefficient for β_1, but in generical the sign and value of this estimate is unclear. In fact, for certain network structures the individual coefficients may average to zero, leading one to falsely reject the Townsend test (even when controlling for appropriate local aggregates) because the underlying heterogeneity is not properly accounted for.

Finally, notice that under sufficiently symmetric structures, we cannot reject this localized version of the Townsend test, because in "regular" networks $\alpha_{ii} - \alpha_{ij} = 0$. We don't consider these cases a false rejection of the "local Townsend test" since, for these structures, once one controls for local aggregates, individual income indeed does not explain consumption any longer. This means we are able to generalize the discussion on appropriate local aggregates in Townsend regressions – the theory is sufficiently rich to accommodate previous models of local sharing groups, as well as many other local structures. In fact, a well-defined local version of the Townsend test may fail to reject full insurance not only if castes or extended families are perfectly connected partitions (as stressed in the previous literature), but also if the social structure is sufficiently symmetric. For an extreme example, consider the circle network in which all individuals are identically positioned. Although all local sharing groups overlap and none of them are perfectly connected, this network structure would nonetheless generate sufficient regularity to "pass" an appropriately defined version of the risk-sharing test.

[Table 1 about here.]

The previous discussion can be observed compactly in Table 1, where the risk-sharing test is performed on simulated income data for the three individual "star" network of section 2, and the four individual "circle" network that exhibits perfect symmetry.[33] The test is performed both with a common aggregate income term (columns 1 and 3) and with appropriately defined local sharing groups (columns 2 and 4). Notice that coefficients on own income are biased upwards by a whole order of magnitude when imposing a common aggregate income term but remain positive and significant under local aggregate in the star network, where the lack of symmetry keeps the pooled coefficient estimate away from zero. However, as discussed above, the circle network "passes" the Townsend test (coefficient to income is not significant) under appropriately specified local aggregate income terms.[34]

6 Extensions

6.1 Within the CARA-Normal Setting

We first consider extensions of our main model *within* the CARA-Normal setting.

6.1.1 Heterogeneity in Risk Aversion and Endowment Distribution

In Section 3.3.1, we assumed that individuals have i.i.d. endowment distribution $e_i \sim_{iid} \mathcal{N}(0, \sigma^2)$ and CARA utility function $u_i(x) = -\exp(-rx)$ with a homogeneous risk-aversion parameter

[33] In both simulations, we randomly draw the endowment $e_i^{(t)}$ of each household from an independent standard normal distribution for $T = 5000$ times. We compute consumption according to our model with an additive error term and regress on both global and local aggregates, controlling for household fixed effects that capture expected transfers.

[34] Note, however, that this result is driven by the perfect symmetry of the circle network. In other simulation with "bow-tie" or kite-shaped networks, we find that the estimated β_1 from a pooled regression with local aggregate income as control can be positive, negative, or very close to zero, while the true underlying "$\beta_i' s$" implied by the PE transfer shares in our model is heterogeneous across individuals and significantly different from zero.

$r > 0$. We then derived the Pareto efficiency of the *local equal sharing rule* under this setting in Proposition 3.

We now establish in the following proposition that rich individual heterogeneity in the endowment distribution and the risk-aversion parameter *within* the CARA-normal setting can be easily incorporated.

Proposition 6. *Suppose that: (1) each individual i's utility function is given by $u_i(x) = -\exp(-r_i x)$ for some individual-specific risk-aversion parameter $r_i > 0$; (2) each individual i's random endowment $y_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$ for some individual-specific mean μ_i and variance σ_i^2; (3) endowments y_i are independent across individuals. Then the* ***weighted local equal sharing rule*** *of the following form*

$$t_{ij}^* := \frac{r_j^{-1}}{\sum_{k \in \overline{N}_i} r_k^{-1}} \cdot y_i - \frac{r_i^{-1}}{\sum_{k \in \overline{N}_j} r_k^{-1}} \cdot y_j + c_{ij} \quad \text{for some constant } c_{ij},$$

is Pareto efficient subject to the local information constraints.

We now discuss the intuitions underlying Proposition 6:

First observe that the transfer shares in Proposition 6 depend only on the risk-aversion parameters r_i but not on the means and variances of the endowment distributions. In particular, whenever risk-aversion parameters are homogeneous, i.e., $r_i = r$, the weighted local equal sharing rule specializes to the local equal sharing rule as in Proposition 3.

It should be clear under the CARA-normal setting that the levels of expected endowments μ_i will have *no* impact on the Pareto efficiency, since any heterogeneous μ_i can be perfectly absorbed into constant transfers c_{ij}, which is completely irrelevant to Pareto efficiency. Hence, the specification of zero mean in Section 3.3.1 should be considered as a normalization without loss of generality rather than a restrictive assumption.[35]

Moreover, the individual-specific variance σ_i^2 does not affect transfer shares neither. To see the underlying intuition, notice that under the local information constraints, individual i's endowment shock can only be shared by i and i's d_i neighbors in $\overline{N}_i$, regardless of how large the variance of i's endowment is. Assuming for simplicity that the risk-aversion parameters are homogeneous ($r_i = r$), then intuitively the best thing to distribute i's endowment shock is to ask i and i's neighbors to equally share i's income shock, regardless of σ_i^2. For those individuals who end up being exposed to more volatile endowment shocks, they can be again compensated via constant transfers.

[35]Consequently, the specification that final consumption can take negative values is not as pathological as it appears at first sight. Instead, a negative amount of final consumption for individual i in Section 3.3 should be interpreted relative to any (positive) *expected* level of endowment μ_i.

In contrast, heterogeneity in the absolute risk-aversion parameters r_i does affect the Pareto efficient transfer shares, and it induces a natural adaption of the local equal sharing rule. Specifically, i's endowment y_i is still shared within i's neighborhood, but the share of y_i undertaken by each individual $k \in \overline{N}_i$ is *inversely proportional* to individual k's risk-aversion parameter r_k. Intuitively, it is less costly for a less risk-averse individual to undertake risk, so Pareto efficiency dictates that a less risk-averse individual undertake larger shares of risky endowment shocks relative to others.

Lastly, Proposition 6 is complementary to our theoretical result on the positive relationship between network centrality and consumption variance in Section 4. In real-world risk-sharing networks, heterogeneity in risk aversion is likely to be present, and individuals' risk preferences may actually be correlated with network centrality. In fact, it could be argued that individuals with *higher* network degrees are likely to be *less* risk-averse in many settings. Under such settings, Proposition 6 suggests that the positive correlation between degree centrality and consumption variance as derived in Section 4 should be even more positive.

6.1.2 Spatial Correlation Structure

The CARA-normal setting we considered earlier features a uniform global correlation structure, in which the correlation between the endowments of two individuals did not depend on their positions on the network. An alternative specification, however, is to incorporate the possibility of spatially correlated endowments, that is correlation that decays with social distance.[36] As we illustrate below (and in more details in Appendix B.9), this type of correlation structure can be detrimental to the efficiency of informal risk sharing with local information constraints.

For concreteness, take the same environment as in Section 3.3 (identical CARA utilities and jointly normally distributed endowments), but assume that the correlation between e_i and e_j geometrically decays with the social distance between i and j : $Corr\,(e_i, e_j) = \varrho^{dist(i,j)}$, where the social distance $dist\,(i,j)$ is formally defined as the length (i.e., the number of links) of the shortest path connecting i and j in network G. Also, for analytical simplicity we focus on circle networks with $n = 2m + 1$ individuals. Under this setting, the following linear transfer arrangements can be shown to be Pareto efficient:

$$t^*_{ij}\,(e_i, e_j) = \frac{1}{3-\varrho} \cdot e_i - \frac{1}{3-\varrho} \cdot e_j$$

In order to make comparable the risk-sharing efficiencies under geometrically decaying spa-

[36]There are many reasons why this correlation structure is more realistic for certain types of endowment shocks: for example, as shown in Fafchamps and Gubert (2007) and in Conley and Udry (2010), social distance tends to be highly correlated with geographic proximity.

tial correlation structure with that under the uniform global correlation structure analyzed in Section 3.3, we control the "shareable risk" to be the same across the two specifications by setting $\rho = \rho_m(\varrho) := \frac{\varrho(1-\varrho^m)}{m(1-\varrho)}$, where ρ is the uniform global pairwise correlation, while ϱ is the rate of decay in the geometrically decaying correlation structure. Then informal risk sharing subject to the local information constraint achieves drastically different levels of asymptotic efficiency under the two correlation structures.

Proposition 7. *Let $x_i^{unif}(\rho), x_i^{geo}(\varrho)$ denote the Pareto efficient consumption plan subject to the local information constraint under the uniform and the geometrically decaying correlation structures, parameterized by ρ and ϱ respectively, and let $Var_{unif,\rho}$, $Var_{geo,\rho}$ correspond to the variance operators under the two probability distributions induced by the two correlation structures. Then:*

$$\lim_{\varrho\to 1}\lim_{m\to\infty} Var_{unif,\rho_m(\varrho)}\left(x_i^{unif}(\rho_m(\varrho))\right) = \frac{1}{3},$$

$$\lim_{\varrho\to 1}\lim_{m\to\infty} Var_{geo,\rho}\left(x_i^{geo}(\varrho)\right) = 1.$$

Hence, for ϱ close to 1 and sufficiently large m, uniform correlation leads to significant risk sharing (yielding payoffs close to that under independent endowments), while geometrically decaying correlation yields payoffs very close to the autarky payoffs, even though the two correlation structures lead to the same payoffs if global information can be used for risk sharing.

This difference in risk-sharing efficiency, driven by the difference in underlying correlation structures, is a peculiar feature of the local information constraint considered in this paper. With global information, a geometrically decaying correlation structure does not in itself imply risk-sharing inefficiency relative to the uniform correlation structure. For example, in a large ring network considered above, shocks that are spatially far away from each other are almost independent, and each given individual is spatially far away from most of the individuals in the network. Hence, under global information mostly shocks with low correlations are pooled together, thus yielding significant risk reduction. However, with local information, only spatially close shocks are pooled, rendering risk sharing virtually ineffective due to the high local correlation.

This might help explain why it is the case that while in most settings empirical research found that informal insurance works well, Kazianga and Udry (2006) found a setting in which informal insurance does not seem to help, and Goldstein, de Janvry, and Sadoulet (2001) found that certain types of endowment shocks are not well insured through informal risk sharing. In particular, this may be due to high correlation between endowments of neighboring households in the above settings, for the types of endowment shocks investigated.

6.1.3 General Correlation Structure

Under the CARA-normal setting with a general correlation structure $\mathcal{N}(\mathbf{0}, \Sigma)$, the conditional distribution of nonlocal e_k given the local information set I_{ij} is given by

$$e_k|_{I_{ij}} \sim \mathcal{N}\left(\Sigma_{k,\overline{N}_{ij}} \Sigma^{-1}_{\overline{N}_{ij}} e_{\overline{N}_{ij}},\ \Sigma_{kk} - \Sigma_{k,\overline{N}_{ij}} \Sigma^{-1}_{\overline{N}_{ij}} \Sigma_{\overline{N}_{ij},k}\right),$$

which is a generalization of (7). Notice that, as in (7), the conditional expectation of e_k given I_{ij} is linear in $e_{\overline{N}_{ij}}$, while the conditional variance is a constant. We can again postulate a linear transfer rule and obtain a linear system in the style of (10), though the algebra will be much more complicated.

Assuming that the Pareto efficient consumption plan can still be achieved by a *linear* profile of transfer arrangements, we can again equivalently solve for the optimal transfer shares by minimizing the sum of consumption variances as in 11:

$$\min_{\alpha} \sum_{i \in N} Var\left[\left(1 - \sum_{j \in N_i} \alpha_{ij}\right) e_i + \sum_{j \in N_i} \alpha_{ji} e_j\right] \equiv \sum_{i \in N} \alpha'_{\to i} \Sigma \alpha_{\to i} \tag{19}$$

where the "exposure shares" $\alpha_{\to i} := (\alpha_{ji})_{j \in N}$ with $\alpha_{ii} := 1 - \sum_{k \in N_i} \alpha_{ik}$ for all i and $\alpha_{ji} = 0$ whenever $G_{ij} = 0$. The above then becomes a well-defined optimization problem with linear constraints, and there are standard computational algorithms to solve this problem numerically. Hence, even though it would be algebraically cumbersome to work out the analytical formula for the Pareto efficient transfer shares under a general correlation structure, (19) provides a numerical guideline to solve for the Pareto efficient transfer arrangements, demonstrating the applicability of our results in more general environments than the base model.

6.2 Beyond the CARA-Normal Setting

We now offer some discussions on the generality of our framework and the robustness of our analytical results beyond the CARA-Normal setting.

6.2.1 Quadratic Utility Function

Quadratic utility functions of the form $u_i(x_i) = x_i - \frac{1}{2} r x_i^2$ for $i \in N$ have also been widely used in the network theory literature due to the analytical tractability it affords. Similar to CARA utility functions, quadratic utility functions also the mean-variance expected utility representation. Noting that $u_i'(x_i) = 1 - r x_i$, the conditional Borch rule in Proposition 1 takes the form of $\lambda_i (1 - r\mathbb{E}_{ij}[x_i]) = \lambda_j (1 - r\mathbb{E}_{ij}[x_j])$. With equal Pareto weightings ($\lambda = \mathbf{1}$) and elliptical distribution of endowments,[37] we show in Appendix B.10 that this leads to exactly the same system of linear equations as in (26), so that Proposition 4 applies without change. However, it should be pointed out that the Pareto efficient frontier traced out by all admissible Pareto weightings will correspond to a collection of different state-dependent transfer shares α.

6.2.2 Edge-Regular Networks

With general utility function and general endowment distribution, we now show that the *local equal sharing rule*, as characterized in Proposition 3 in the form of $t_{ij}^*(I_{ij}) = \frac{1}{d+1}e_i - \frac{1}{d+1}e_j + \mu_{ij}$, remains Pareto efficient under certain symmetry conditions. We focus our attention on the local equal sharing rule due to its particular importance in this paper: it is not only the key components of Propositions 3 and 4 in Section 3.3, but also the underlying transfer rules we used in Sections 4 and 5.

Specifically, a network G is said to be *edge-regular network* with parameters (N, d, λ), if each of the N individuals in the network has d neighbors and each linked pair of individuals share λ common neighbors. Notice that a circle network with N individuals is a special case of edge-regular network with parameters $d = 2$ and $\lambda = 0$.

Proposition 8. *Suppose that G is an edge-regular network. Let y_i be i.i.d. endowments with any distribution and u be any concave utility function. Write $\mu := \mathbb{E}[y_i]$ and write $e_i = y_i - \mu$. Then the local equal sharing rule $t_{ij}^*(I_{ij}) = \frac{1}{d+1}e_i - \frac{1}{d+1}e_j$ is Pareto efficient.*

The proof follows immediately from the observation that, under the local equal sharing rule t^*, individual i's final consumption

$$x_i^* = \mu + \frac{1}{d+1}\sum_{k\in\overline{N}_{ij}} e_k + \frac{1}{d+1}\sum_{k\in N_i\setminus\overline{N}_j} e_k$$

has exactly the same conditional distribution as neighbor j's final consumption x_j^* given any local state I_{ij}, ensuring that the local Borch rule be satisfied.

6.2.3 Approximate Pareto Efficiency

In more general settings, the local equal sharing rule may not be exactly Pareto efficient. However, we show in this subsection that the local equal sharing rule "satisfies the first-order

[37] In fact, we only need that the joint distribution of endowments satisfies the linear conditional expectatation (LCE) property, which is satisfied by the elliptical family of distributions. Note that the multivariate normal distribution belongs to the elliptical family.

Taylor terms" of the local Borch rule, and hence can be sometimes regarded as *approximately* Pareto efficiency.

Specifically, suppose that endowments y_i are iid with $\mathbb{E}[y_i] = \mu$ and utility functions are homogeneous $u_i = u$. Again write $e_i := y_i - \mu$ as the endowment shock. Under the local equal sharing rule individual i's final consumption can be decomposed as

$$x_i = x_i(I_{ij}) + \xi_{N_i \backslash \overline{N}_j} \quad \text{s.t.} \quad \mathbb{E}[\xi_{N_i \backslash \overline{N}_j} | I_{ij}] = 0,$$

with $x_i(I_{ij}) := \mu + \frac{1}{d+1} \sum_{k \in \overline{N}_{ij}} e_k$ and $\xi_{N_i \backslash \overline{N}_j} := \sum_{k \in N_i \backslash \overline{N}_j} \frac{1}{d_k+1} e_k$.

Recall that the local Borch rule requires (WLOG under equal Pareto weights) that, for Pareto efficiency, $\mathbb{E}\left[u'(x_i) \middle| I_{ij}\right] = \mathbb{E}\left[u'(x_j) \middle| I_{ij}\right]$, or equivalently

$$\mathbb{E}\left[u'\left(x_i(I_{ij}) + \xi_{N_i \backslash \overline{N}_j}\right) \middle| I_{ij}\right] = \mathbb{E}\left[u'\left(x_j(I_{ij}) + \xi_{N_j \backslash \overline{N}_i}\right) \middle| I_{ij}\right],$$

which is in general not satisfied as the distribution of $\xi_{N_i \backslash \overline{N}_j}$ can be different from the distribution of $\xi_{N_j \backslash \overline{N}_i}$.

Writing $\overline{I}_{ij}$ as a generic realization of I_{ij}, we take the following standard Taylor expansion of the conditional expected marginal utility around $x_i(I_{ij})$:

$$\mathbb{E}\left[u'(x_i) \middle| I_{ij}\right] = \mathbb{E}\left[u'\left(x_i\left(\overline{I}_{ij}\right)\right)\right] + \frac{1}{2}\mathbb{E}\left[u'''\left(x_i\left(\overline{I}_{ij}\right) + \tilde{\xi}_{N_i \backslash \overline{N}_j}\right)\xi^2_{N_i \backslash \overline{N}_j}\right] \tag{20}$$

where $\tilde{\xi}_{N_i \backslash \overline{N}_j}$ is some random variable that lies between 0 and $\xi_{N_i \backslash \overline{N}_j}$.

The local equal sharing rule guarantees that $x_i\left(\overline{I}_{ij}\right) \equiv x_j\left(\overline{I}_{ij}\right)$, so the first term on the right-hand side of (20) is equalized. Hence, the local Borch rule can be said to hold *approximately* provided that the second higher-order terms in (20) are relatively small. Clearly, this approximation will be relatively good when u''' is reasonably small around $x_i\left(\overline{I}_{ij}\right)$ or when the non-local shock exposures $\xi_{N_i \backslash \overline{N}_j}$ and $\xi_{N_j \backslash \overline{N}_i}$ are relatively small.

We show more explicitly in Appendix B.11 that the local equal sharing is approximately Pareto efficient in (the very asymmetric) star networks with a CRRA utility function $u(x) = \log(x)$ and uniform distribution of endowments, a setting where the exactly Pareto efficient transfer rules *cannot* be linear. Also, we provide numerical illustration of the Pareto efficiency of the local equal sharing rule when endowments follow log-normal or Bernoulli distributions. Despite the peculiarity of these illustrations, they show that the local equal sharing rule can be relevant in a variety of cases beyond the CARA-normal setting.

7 Conclusion

This paper analyzes informal risk sharing arrangements under local information constraints, when bilateral transfers can only depend on endowment realizations of a subset of individuals. We characterize the Pareto efficient consumption allocations in this setting, and provide closed-form descriptions of the bilateral transfer arrangements that lead to them in a widely studied context of CARA utilities and jointly normally distributed endowments. We show that more central individuals have more volatile consumption and we test this implication using data from rural villages in Thailand. This model generalizes the notion of a local sharing group that has been invoked recently in the risk-sharing tests performed in the development literature.

The model provides numerous further implications for empirical work. In a first approach, Milán et al. (2018) show that the current framework fits the observed sharing behavior of indigenous communities in the Bolivian Amazon. However, further empirical work is needed to distinguish local information constraints from other similar contractual frictions, such as the hidden income model identified by Kinnan (2017) as the relevant friction in Thai data. Indeed, in future work we plan to derive a dynamic version of the model that provides testable predictions between current consumption and past information, which can be compared to those of other proposed risk-sharing frictions. Another potential empirical project building on the current work would take the model's predictions on bilateral exchanges in order to develop a complete model of spillover effects across individuals that can be used to structurally estimate the underlying network structure following techniques in Manresa (2016) and most recently in De Paula, Rasul, and Souza (2018) .

References

Alatas, V., A. Banerjee, A. G. Chandrasekhar, R. Hanna, and B. A. Olken (2016): "Network structure and the aggregation of information: Theory and evidence from Indonesia," *American Economic Review*, 106, 1663–1704.

Ambrus, A. and M. Elliott (2020): "Investments in Social Ties, Risk Sharing and Inequality," Working Paper.

Ambrus, A., M. Mobius, and A. Szeidl (2014): "Consumption Risk-Sharing in Social Networks," *The American Economic Review*, 104, 149–182.

Angelucci, M., G. De Giorgi, and I. Rasul (2017): "Consumption and investment in resource pooling family networks," *The Economic Journal*, doi:10.1111/ecoj.12534.

Attanasio, O., C. Meghir, and C. Mommaerts (2018): "Insurance in Extended Family Networks," Cowles Foundation Discussion Paper No. 1996R.

Banerjee, A., A. G. Chandrasekhar, E. Duflo, and M. O. Jackson (2018): "Using gossips to spread information: theory and evidence from two randomized controlled trials," NBER Working Paper No. 20422.

Belhaj, M. and F. Deroïan (2012): "Risk taking under heterogenous revenue sharing," *Journal of Development Economics*, 98, 192–202.

Bloch, F., G. Genicot, and D. Ray (2008): "Informal Insurance in Social Networks," *Journal of Economic Theory*, 143, 36–58.

Bloch, F. and M. Olckers (2018): "Friend-based Ranking," Working Paper.

Borch, K. (1962): "Application of Game Theory to Some Problems in Automobile Insurance," *Astin Bulletin*, 2, 208–221.

Bourlès, R., Y. Bramoullé, and E. Perez-Richet (2017): "Altruism in networks," *Econometrica*, 85, 675–689.

——— (2018): "Altruism and risk sharing in networks," .

Bramoullé, Y. and R. Kranton (2007): "Risk-Sharing Networks," *Journal of Economic Behavior & Organization*, 64, 275–294.

Chiappori, P.-A., K. Samphantharak, S. Schulhofer-Wohl, and R. M. Townsend (2014): "Heterogeneity and risk sharing in village economies," *Quantitative Economics*, 5, 1–27.

Coate, S. and M. Ravallion (1993): "Reciprocity without Commitment: Characterization and Performance of Informal Insurance Arrangements," *Journal of Development Economics*, 40, 1–24.

Cochrane, J. (1991): "A simple test of consumption insurance," *Journal of Political Economy*, 99, 957–976.

Conley, T. G. and C. R. Udry (2010): "Learning about a New Technology: Pineapple in Ghana," *The American Economic Review*, 100, 35–69.

De Paula, Á., I. Rasul, and P. Souza (2018): "Recovering social networks from panel data: Identification, simulations and an application," Cemmap Working Paper CWP17/18.

Deaton, A. (1992): "Household Saving in LDCs: Credit Markets, Insurance and Welfare," *The Scandinavian Journal of Economics*, 94, 253–273.

Dubois, P., B. Jullien, and T. Magnac (2008): "Formal and informal risk sharing in LDCs: Theory and empirical evidence," *Econometrica*, 76, 679–725.

Eaton, M. L. (2007): *Multivariate statistics: a vector space approach*, Institute of Mathematical Statistics.

Ellsworth, L. (1988): "Mutual insurance and non-market transactions among farmers in Burkina Faso," Ph.D. thesis, University of Wisconsin, Madison.

Fafchamps, M. (1999): "Risk sharing and quasi-credit," *Journal of International Trade and Economic Development*, 8, 257–278.

Fafchamps, M. and F. Gubert (2007): "The formation of risk sharing networks," *Journal of Development Economics*, 83, 326–350.

Fafchamps, M. and S. Lund (2003): "Risk-sharing networks in rural Philippines," *Journal of Development Economics*, 71, 261–287.

Gao, W. Y. and E. Moon (2016): "Informal Insurance Networks," *The B.E. Journal of Theoretical Economics*, 16, 455–484.

Goldstein, M., A. de Janvry, and E. Sadoulet (2001): "You can't always get what you want: Inclusion and Exclusion in Mutual Insurance Networks in Southern Ghana," .

Grimard, F. (1997): "Household consumption smoothing through ethnic ties: evidence from Cote d'Ivoire," *Journal of Development Economics*, 53, 391–422.

Hayashi, F., J. Altonji, and L. Kotlikoff (1996): "Risk-Sharing between and within Families," *Econometrica*, 64, 261–294.

Jackson, M. O. and A. Wolinsky (1996): "A strategic model of social and economic networks," *Journal of Economic Theory*, 71, 44–74.

Karaivanov, A. and R. M. Townsend (2014): "Dynamic financial constraints: Distinguishing mechanism design from exogenously incomplete regimes," *Econometrica*, 82, 887–959.

Karlan, D., M. Mobius, T. Rosenblat, and A. Szeidl (2009): "Trust and social collateral," *The Quarterly Journal of Economics*, 124, 1307–1361.

Kazianga, H. and C. Udry (2006): "Consumption smoothing? Livestock, insurance and drought in rural Burkina Faso," *Journal of Development Economics*, 79, 413–446.

Kinnan, C. (2017): "Distinguishing barriers to insurance in Thai villages," Mimeo Northwestern University.

Kocherlakota, N. R. (1996): "Implications of efficient risk sharing without commitment," *The Review of Economic Studies*, 63, 595–609.

Ligon, E. (1998): "Risk sharing and information in village economies," *The Review of Economic Studies*, 65, 847–864.

Ligon, E., J. P. Thomas, and T. Worrall (2002): "Informal insurance arrangements with limited commitment: Theory and evidence from village economies," *The Review of Economic Studies*, 69, 209–244.

Mace, B. (1991): "Full Insurance in the Presence of Aggregate Uncertainty," *Journal of Political Economy*, 99, 928–956.

Manresa, E. (2016): "Estimating the Structure of Social Interactions Using Panel Data," Working Paper.

Mazzocco, M. and S. Saini (2012): "Testing Efficient Risk Sharing with Heterogeneous Risk Preferences," *American Economic Review*, 102, 428–68.

Milán, P., M. Gurven, P. Hooper, and H. Kaplan (2018): "Partial Insurance Networks: Food Sharing in Tsimane' Villages," Working paper, Barcelona GSE.

Munshi, K. and M. Rosenzweig (2016): "Networks and Misallocation: Insurance, Migration, and the Rural-Urban Wage Gap," *American Economic Review*, 106, 46–98.

Myerson, R. B. (1980): "Conference structures and fair allocation rules," *International Journal of Game Theory*, 9, 169–182.

——— (1991): *Game theory: analysis of conflict*, Cambridge,MA: Harvard University.

Paravisini, D., V. Rappoport, and E. Ravina (2016): "Risk aversion and wealth: Evidence from person-to-person lending portfolios," *Management Science*, 63, 279–297.

Paxson, C. H. (1993): "Consumption and income seasonality in Thailand," *Journal of Political Economy*, 101, 39–72.

Peypouquet, J. (2015): *Convex optimization in normed spaces: theory, methods and examples*, Springer.

Ravallion, M. and S. Chaudhuri (1997): "Risk and insurance in village India: Comment," *Econometrica*, 65, 171–184.

Rosenzweig, M. R. (1988): "Risk, implicit contracts and the family in rural areas of low-income countries," *The Economic Journal*, 98, 1148–1170.

Schulhofer-Wohl, S. (2011): "Heterogeneity and Tests of Risk Sharing," *The Journal of Political Economy*, 119, 925–958.

Shrinivas, A. and M. Fafchamps (2018): "Testing Efficient Risk Sharing with Heterogeneous Risk Preferences: Comment," *American Economic Review*, 108, 3104–13.

Stole, L. A. and J. Zwiebel (1996): "Intra-firm bargaining under non-binding contracts," *The Review of Economic Studies*, 63, 375–410.

Suri, T. (2005): "Spillovers in village consumption: Testing the extent of partial insurance," *New Haven, Conn.: Yale University. mimeo.*

Thomas, J. and T. Worrall (1990): "Income fluctuation and asymmetric information: An example of a repeated principal-agent problem," *Journal of Economic Theory*, 51, 367–390.

Townsend, R. M. (1982): "Optimal multiperiod contracts and the gain from enduring relationships under private information," *The Journal of Political Economy*, 90, 1166–1186.

——— (1994): "Risk and insurance in village India," *Econometrica*, 62, 539–591.

Udry, C. (1994): "Risk and insurance in a rural credit market: An empirical investigation in northern Nigeria," *The Review of Economic Studies*, 61, 495–526.

Wang, C. (1995): "Dynamic insurance with private information and balanced budgets," *The Review of Economic Studies*, 62, 577–595.

Wilson, R. (1968): "The theory of syndicates," *Econometrica*, 36, 119–132.

Table 1: Simulated Risk-Sharing Test under the Model for Two Simple Economies

	Dependent variable: Consumption			
	Star Network		*Circle Network*	
	Common Group	Local Group	Common Group	Local Group
	(1)	(2)	(3)	(4)
Income	0.173	0.026	0.109	0.002
	(0.003)	(0.002)	(0.003)	(0.002)
Agg. Income	0.276	0.419	0.223	0.332
	(0.002)	(0.001)	(0.001)	(0.001)
Observations	15,000	15,000	20,000	20,000
R^2	0.755	0.923	0.716	0.916

Note: Values in parentheses are standard errors.

Supplement to "Informal Risk Sharing with Local Information"

Attila Ambrus[38], Wayne Y. Gao[39], Pau Milán[40]

A Other Major Extensions

A.1 General Contractibility Constraints

In Section 2.4 we showed – for a simple line network – how to describe risk-sharing arrangements subject to local information constraints that need not coincide with the physical transfer network. We now return to this question and provide a general characterization of the admissible structures where our main results continue to apply.

As before, let G denote a generic undirected and unweighted network structure defined on N. We now interpret G as *physical (transfer) network*: two individuals i and j can enter into a risk-sharing transfer contract if and only if they are linked in G, or $G_{ij} = 1$. As before, N_i and N_{ij} denote i's neighborhood and ij's common neighborhood under G, respectively.

We now specify a more general form of contractibility constraints. Suppose now that, for each linked pair of individuals ij in G, their bilateral transfer contract t_{ij} can be (effectively) contingent on the ex post realizations of the income shocks of individuals in some predetermined set $Q_{ij} \subseteq N \setminus \{i, j\}$, in addition to their own income shocks e_i and e_j. In other words, t_{ij} can be contingent on the ex post realizations of e_k for all $k \in \overline{Q}_{ij} := \{i, j\} \cup Q_{ij}$. We write Q (and equivalently $\overline{Q}$) to denote the joint requirements of pairwise contractibility constraints Q_{ij} for all linked individuals in G.[41]

Clearly, by taking $Q_{ij} = N_{ij}$ for all linked ij, we reduce the model back to the special case of local information constraints as formalized in Subsection 3.1. By taking $Q_{ij} = N \setminus \{i, j\}$ for all ij, we reduce the model back to the simple "global-information" benchmark.

[38] Ambrus: Department of Economics, Duke University, Durham, NC 27708, USA, attila.ambrus@duke.edu.
[39] Gao: Department of Economics, University of Pennsylvania, Philadelphia, PA 19104, USA, waynegao@sas.upenn.edu.
[40] Milán: Department of Economics, Universitat Autònoma de Barcelona, Edifici B, Bellaterra 08193, Spain, pau.milan@uab.cat.
[41] Alternatively, we could specify Q without reference to G. However, as we take both G and Q as primitives, this expositional difference is inconsequential.

In this subsection we take Q as the primitive, and discuss how our methods and results can be adapted to accommodate the contractibility constraints encoded by Q. The question how the contractibility constraints Q may arise from individuals' ex-post interactions that may support on-equilibrium information transmission will be deferred to the next subsection.

Clearly, under general contracting constraints encoded by Q, the social planner's problem 3 remains a convex optimization problem: the objective function remains concave, while the choice space (space of admissible transfer arrangements under G and Q) remains a convex set.

Corollary 2. *Propositions 1 and 2 carry over with proper notational adaptions.*

Consequently Corollary 1 (the localized Borch rule) remains valid, too, with the conditional expectations on the left hand side of $\frac{\mathbb{E}_{ij}\left[u_i'\left(x_i^t\right)\right]}{\mathbb{E}_{ij}\left[u_j'\left(x_j^t\right)\right]} = \frac{\lambda_j}{\lambda_i}$ being taken with respect to the more general local information sets.

Next, we again specialize to the CARA-normal setting as considered in Subsection 3.3. We first provide a sufficient condition under which Propositions 3 and 4 generalize almost exactly.

Proposition 9. *Suppose that G is connected as before and that there exists an undirected and unweighted supergraph of G, denoted G', such that:*

(a) The contractibility constraints Q satisfies that $Q_{ij} = N_i' \cap N_j'$ for all linked ij in the original network G, where N_i' denotes i's neighborhood in the supergraph network G'.

(b) For every pair ij linked in G', there exists a path in G from i to j such that, for any individual k that lies on this path, we have that ik and jk are also linked in G'.

Then the constrained Pareto efficient consumption plan under (G, Q) is given by the consumption plan $x^*\left(G'\right)$ induced by the hypothetical linear transfer rules $t^*\left(G'\right)$, or equivalently the transfer shares $\alpha^*\left(G'\right)$, as defined in Propositions 3 and 4.

Condition (a) essentially requires that all contractibility constraints are induced by common neighborhoods under an "informational network" G' that is a supergraph of the physical transfer network G. Condition (b) essentially requires that the physical transfer network G is rich enough to channel, potentially via a path of individuals in G, any net bilateral transfer scheme between two informationally linked individuals in G'. Simple examples of G' that satisfies condition (b) includes a supergraph of G which add (informational) links between some distance-2 pairs of individuals in the physical network G, and a supergraph of G which add links between all individuals within a distance of k from each other in the physical network G.[42]

[42]See the next subsection for a concrete example of how communication among individuals can establish such informational networks.

Under conditions $(a)(b)$, only the "informational network" G' is relevant in determining the constrained Pareto efficient consumption plan, or equivalently the risk sharing transfer arrangements up to superfluous cyclical transfers, which can be computed by exactly the same formulas given by Propositions 3 and 4 with the informational network G' as the relevant network structure.

In the next subsection (A.2), we provide several examples of realistic ex-post communication protocols that may give rise to contractibility constraints Q that satisfies Conditions (a) and (b), so that it is sufficient to focus on the "informational network" G'.

Before proceeding to the next subsection, we point out that the method of analysis, together with some particular results, generalizes beyond Proposition 9. For simplicity, in the following we focus on the case of independent endowments ($\rho = 0$), and provide sufficient conditions under which *the local equal sharing rule* generalizes.

Proposition 10. *Let $\rho = 0$. Given a network G and contractibility constraints Q, define a directed network $\overleftarrow{G}$ by setting $\overleftarrow{G}_{ij} = 1$ if and only if there exists a path of individuals $i = k_0 k_1 ... k_m = j$ in G, such that $j \in Q_{k_h k_{h+1}}$ for all $h = 0, ..., m-1$. Define the "in-neighborhood" $N_i\left(\overleftarrow{G}\right) := \{k \in N : G_{ij} = 1\}$ and the "in-degree" $d_i\left(\overleftarrow{G}\right) := \#\left(N_i\left(\overleftarrow{G}\right)\right)$ accordingly. Then the following consumption plan*

$$x_i^*\left(\overleftarrow{G}\right) := \frac{1}{d_i\left(\overleftarrow{G}\right)+1} e_i + \sum_{j \in N_i\left(\overleftarrow{G}\right)} \frac{1}{d_j\left(\overleftarrow{G}\right)+1} e_j \tag{21}$$

is constrained Pareto efficient subject to the contractibility constraints Q under network G.

Compared to Proposition 9, Proposition 10 relaxes both Condition (a) and Condition (b). Specifically, Q is no longer restricted to be inducible as common neighborhoods of an undirected graph, allowing for scenarios where $k \in Q_{ij}$ but $i \notin Q_{jk}$. Correspondingly, the constructed "informational network" $\overleftarrow{G}$ is directed, only requiring that information about individual j's endowment realization can transmit to individual i, but not necessarily vice versa. Most importantly, Proposition 10 asserts that, for any $k \in G_{ij} \backslash \overleftarrow{N}_{ij}$, which may not be empty in general, the Pareto efficient consumption for ij, $x_i^*\left(\overleftarrow{G}\right)$ and $x_j^*\left(\overleftarrow{G}\right)$, are necessarily independent of e_k, even though t_{ij} may be made contingent on e_k. This feature is specific to the case of independent endowments, which is not covered by Proposition 9.

A.2 Risk Sharing with Ex-Post Communication

In Section A.1, we abstract from the detailed specification of such ex post interactions, but instead use the contractibility constraints Q as a reduced-form representation of ex post interactions on the effective contractibility of risk sharing arrangements. In the current section, we investigate particular detailed specifications of ex post interactions. We show that natural specifications produce a contractibility structure Q as specified in Section A.1 that satisfies both Conditions (a) and (b) in Proposition 9, and hence we may compute the constrained Pareto efficient risk-sharing arrangements by directly applying Proposition 4 using the relevant *informational network* G'.

Fix any connected network G. Consider a scenario where, after endowment realizations but before transfer payments, a single-round of simultaneous communication is allowed. Specifically, each individual i may send a message $m_{ij} \in \mathscr{M}_{ij}$ to each individual $j \in N \backslash \{i\}$, where $\mathscr{M}_{ij}$ denotes an arbitrary message space. The (local) observability of *messages* is determined by a communication protocol, which we take to be a primitive of the environment. For example, a few simplest communication protocols that lead to different levels of observability of messages are:

(a) *Global communication*: m_{ij} is publicly observable by all individuals Equivalently, we might as well take $m_{ij} \equiv m_i$ and $\mathscr{M}_{ij} \equiv \mathscr{M}_i$, i.e., each individual can only send a public message that then becomes global common knowledge. For example, a global message be thought of as a Tweet, which everyone can observe (if he wants to).

(b) *Local announcement*: m_{ij} is locally observable by the sender i and i's neighbors. Again, we might as well take $m_{ij} \equiv m_i$ and $\mathscr{M}_{ij} \equiv \mathscr{M}_i$. For example, a local announcement can be thought of as a message i posts on his *own* Facebook timeline.

(c) *Local comment*: m_{ij} is locally observable by the receiver j and j's neighbors. For example, a local comment can be thought of as a message i leaves on j's Facebook timeline.

(d) *Private communication*: m_{ij} is only privately observable by the sender i and receiver j. A variety of communication technologies such as personal meeting, phone calls, online chats fit into this category.

Given a communication protocol, for each linked pair ij, their ex post local common knowledge before transfers are carried out not only include the endowment realizations they can commonly observe, denoted $I_{ij} = (e_k)_{k \in \overline{N}_{ij}}$, but also include the communication messages they can commonly observe, denoted M_{ij}, which will differ across the four communication protocols above. Again, we require that the bilateral transfer contract t_{ij} be contingent only on ex post local common knowledge, i.e., t_{ij} be $\sigma(I_{ij}, M_{ij})$-measurable. As before, we abstract from ex post enforcement issues of the contract t_{ij} per se, but focus on the strategic aspects of ex post messages.

We summarize in the following Proposition how the four ex-post communication protocols introduced above may give rise to different *informational network* structures, with which we can directly apply Proposition 9.

Proposition 11. *Under either of the four communication protocols listed above, there exists a profile of bilteral risk-sharing contracts t^* such that: (i) t_{ij} is $\sigma(I_{ij}, M_{ij})$-measurable for each linked ij in G; (ii) there exists an undirected supergraph G' of G such that the constrained Pareto efficient consumption plan $x^*(G')$ with respect to G' can be implemented in ex-post Nash equilibrium; (iii) the effective information network G', under the four communication protocols, is given by, respectively:*

- *(a) Global communication: G' is the complete graph.*
- *(b) Local announcement: $G' = G^{(2)}$, i.e., the graph obtained by linking pairs of individuals within a graphic distance of 2 from each other in the original network G.*
- *(c) Local comment: $G' = G^{(3)}$, i.e., the graph obtained by linking pairs of individuals within a graphic distance of 3 from each other in the original network G.*
- *(d) Private communication: $G' = G$.*

The main idea for constructing the message-augmented contract t^* that induces truthful information transmission in ex-post Nash equilibrium is to *cross-validate* reports of certain non-local endowment realizations from two different individuals. Admittedly, there are clearly many other plausible forms of ex-post interactions that lead to different extents of information transmission, and it is conceivable that some forms of ex-post interactions may not be able to support the "hard-information" contractibility structure as studied in this paper. However, we will defer a more thorough analysis of this problem to future work.[43]

A.3 Endogenous Network Formation

So far our analysis focused on characterizing Pareto efficient risk-sharing arrangements subject to local information constraints on an exogenously given network, implicitly assuming that the

[43] A related exercise is conducted in an ordinal setting in a recent paper by Bloch and Olckers (2018).

network structure is mainly shaped by predetermined factors such as kinship. Here we briefly discuss some implications of allowing for endogenous link formation in the context of informal risk sharing with local information constraints, in the CARA-normal environment of Section 3.3. The approach we take is similar as in Ambrus and Elliott (2020), who consider network formation in a risk-sharing framework with global information contracts, and propose a two-stage game in which in the first stage individuals can simultaneously indicate other individuals they want to link with. If two individuals each indicated each other, the link is formed, and the two connecting individuals each incur a cost of $c \geq 0$.[44] The solution concept we use is pairwise stability. In the second stage, whatever network is formed in the first stage, it is assumed that individuals agree on a Pareto efficient risk-sharing arrangement subject to local information constraints.

In our analysis of the CARA-normal framework so far, state independent transfers played a very limited role. However, when we allow for endogenous network formation, it becomes crucial how the network structure influences state independent transfers, and hence the distribution of surplus created by risk sharing, as it directly affects incentives to form links. Therefore, it is important to specify exactly which Pareto efficient risk-sharing arrangement prevails for each possible network that can form. Different ways of specifying state-independent transfers can lead to very different conclusions regarding network formation, as we demonstrate below.

A benchmark case is when all state-independent transfers are set to 0, which case is extensively investigated by Gao and Moon (2016) who assume local equal sharing with no state-independent transfers as an ad hoc sharing rule. They show that, even with zero cost of linking, an individual i's benefit for establishing an extra link with j falls very fast with the existing number of links the individual i has, as with more existing neighbors (larger d_i) the marginal reduction in self-endowment exposure $\left(\frac{1}{d_i+1} - \frac{1}{d_i+2}\right)$ is small relative to the additional exposure to j's endowment $\frac{1}{d_j+2}$. Typically this implies severe under-investment into social links.

An alternative approach is pursued by Ambrus and Elliott (2020), in the context of risk-sharing arrangements with global information: they assume that the profile of state-independent transfers is determined according to the Myerson value. The Myerson value, proposed in Myerson (1980), is a network-specific version of the Shapley value that allocates surplus according to average incremental contribution of individuals to total social surplus.[45] In particular, Ambrus and Elliott (2020) show that with state-independent transfers specified as above (for whatever

[44]This simple game of network formation was originally considered in Myerson (1991). See also Jackson and Wolinsky (1996).

[45]Ambrus and Elliott (2020) also provide micro-foundations, in the form of a decentralized bargaining procedure between neighboring individuals that leads to state independent transfers achieving the Myerson value allocation.

network is formed), if individuals are ex ante symmetric then there is never under-investment, that is given any stable network, there is no potential link that is not established, even though its net social value would be strictly positive. Below we show that the same conclusion holds in our setting with local information constraints, in the case of CARA utilities and independently an jointly normally distributed endowments. The detailed specification and the proof are available in Appendix C.13.

Proposition 12. *Suppose that, for any given network structure, the Pareto efficient consumption plan subject to the local information constraint is implemented, and the state-independent transfers are induced by the Myerson values. Consider the first-stage network formation game in which each individual pays a private cost of c for each of her established links. Then, there is no under-investment in social links in any pairwise stable network.*

We leave a more detailed investigation of network formation in the context of risk sharing with local information constraints to future research.

B Main Proofs

The proofs for all the lemmas stated in this section are available in Appendix C.

Define $J(t) := \mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k - \sum_{h\in N_k} t_{kh}\right)\right]$, the objective function in equation (3).

Lemma 1. *$\mathcal{T}$ with $\langle\cdot,\cdot\rangle$ forms an inner product space.*

Lemma 2. *J is concave on $\mathcal{T}$.*

Lemma 3. *J is Gâteaux-differentiable.*

Lemma 4. *For any $t \in \mathcal{T}$ that solves (4), we have $J'(t) = \mathbf{0}$.*

Lemma 5. *The set of consumption plan induced by the profiles of transfer rules t in $\mathcal{T}$ is convex.*

B.1 Proof of Proposition 1

Proof. We first prove the "only if" part. Note that, given any $t \in \mathcal{T}^*$, $\forall i, j$,

$$\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k - \sum_{h\in N_k} t_{kh}\right)\right] = \mathbb{E}\left[\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k - \sum_{h\in N_k} t_{kh}\right)\middle| I_{ij}\right]\right]$$
$$\leq \mathbb{E}\left[\max_{t_{ij}\in\mathbb{R}}\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k - \sum_{h\in N_k} t_{kh}\right)\middle| I_{ij}\right]\right]$$

This is because, conditional on I_{ij}, t_{ij} must be constant across all possible states, and thus the maximization of the conditional expectation is to solve for the optimal real number t_{ij}. For t to be a solution for problem (3), suppose there exists linked ij such that t_{ij} does not solve the problem (4). Then, by the inequality above, there exists another t_{ij}, specified for each different realization of I_{ij} and hence each possible state of nature, that leads to higher value of $\mathbb{E}\left[\sum_{k\in N}\lambda_k u_k\left(e_k-\sum_{h\in N_k}t_{kh}\right)\right]$, contradicting the optimality of t for problem (3). Note that the "$\mathbb{P}$-almost-all" quantifier applies here.

For the "if" part, notice that by Lemma 4, t solves all (4) simultaneously implies that $J^{'}(t)=\mathbf{0}$. As $J:\mathcal{T}\to\mathbb{R}$ is concave by Lemma 2 and Gâteaux-differentiable by Lemma 3, we can apply a mathematical result on convex optimization in normed space, specifically Theorem 3.24 and Proposition 3.20 in Peypouquet (2015), to conclude that asserting that if $J^{'}(t)=\mathbf{0}$, then $J(t)$ is the unique global maximum. ∎

B.2 Proof of Proposition 2

Proof. Following the proof of Lemma 2, we can easily show, by the strict concavity of $u_i(\cdot)$, that the objective function in (3) is strictly concave in the consumption plan x. Lemma 5 shows that the set of admissible consumption plan induced by the set of transfer rules in $\mathcal{T}$ is convex. Hence, there is at most of one consumption plan that solves (3) . ∎

B.3 Proof of Corollary 1

Proof. By the concavity (shown in Lemma 2) of the objective function in (4), the FOC is both sufficient and necessary for maximization. The FOC w.r.t t_{ij}, is

$$\mathbb{E}\left[\lambda_i u_i^{'}\left(e_i-\sum_{h\in N_i}t_{ih}(e)\right)+\lambda_j u_j^{'}\left(e_j-\sum_{h\in N_j}t_{jh}(e)\right)\cdot(-1)\middle| I_{ij}\right]=0$$

Rearranging the above we have

$$\frac{\mathbb{E}_{ij}\left[u_i^{'}\left(x_i^t\right)\right]}{\mathbb{E}_{ij}\left[u_j^{'}\left(x_j^t\right)\right]}=\frac{\mathbb{E}\left[u_i^{'}\left(e_i-\sum_{h\in N_i}t_{ih}(e)\right)\middle| I_{ij}\right]}{\mathbb{E}\left[u_j^{'}\left(e_j-\sum_{h\in N_j}t_{jh}(e)\right)\middle| I_{ij}\right]}=\frac{\lambda_j}{\lambda_i}.$$

∎

B.4 Proof of Proposition 3

Proof. Let x_i^* be the consumption plan induced by the transfer t^* described above. Then

$$\begin{aligned}
CE\left(x_i^*|I_{ij}\right) &= \mathbb{E}_{ij}\left[e_i - \sum_{k\in N_i} t_{ik}^*\right] - \frac{1}{2}rVar_{ij}\left[e_i - \sum_{k\in N_i} t_{ik}^*\right] \\
&= e_i - \frac{e_i}{d_i+1} + \frac{e_j}{d_j+1} - \mu_{ij}^* - \sum_{k\in N_{ij}}\left(\frac{e_i}{d_i+1} - \frac{e_k}{d_k+1} + \mu_{ik}^*\right) \\
&\quad - \sum_{k\in N_i\setminus\overline{N}_j}\left(\frac{e_i}{d_i+1} - \frac{\mathbb{E}_{ij}\left[e_k\right]}{d_k+1} + \mu_{ik}^*\right) - \frac{1}{2}rVar\left[\sum_{k\in N_i\setminus\overline{N}_j}\frac{e_k}{d_k+1}\right] \\
&= \frac{e_i}{d_i+1} + \frac{e_j}{d_j+1} + \sum_{k\in N_{ij}}\frac{e_k}{d_k+1} - \sum_{k\in N_i}\mu_{ik}^* - \frac{1}{2}r\sigma^2\cdot\sum_{k\in N_i\setminus\overline{N}_j}\frac{1}{(d_k+1)^2}.
\end{aligned}$$

The necessary and sufficient condition for t^* to be Pareto efficient is given by (6). Plugging the above into (6) and canceling out the terms dependent on local information $(e_k)_{k\in\overline{N}_{ij}}$, we arrive at the following condition for Pareto efficiency:

$$\sum_{k\in N_i}\mu_{ik}^* + \frac{1}{2}r\sigma^2\cdot\sum_{k\in N_i\setminus\overline{N}_j}\frac{1}{(d_k+1)^2} + \frac{1}{r}\ln\lambda_i = \sum_{k\in N_j}\mu_{jk}^* + \frac{1}{2}r\sigma^2\cdot\sum_{k\in N_j\setminus\overline{N}_i}\frac{1}{(d_k+1)^2} + \frac{1}{r}\ln\lambda_j \quad (22)$$

Any profile of state-independent transfers μ^* that solves the above system (22) makes t^* efficient under weightings λ.

Notice that, if $CE\left(x_i^*|\, I_{ij}\right) - \frac{1}{r}\ln\lambda_i = CE\left(x_j^*\middle|\, I_{ij}\right) - \frac{1}{r}\ln\lambda_j$ holds for any e,

$$\begin{aligned}
CE\left(x_i^*\right) - \frac{1}{r}\ln\lambda_i &= \mathbb{E}\left[CE\left(x_i^*|\, I_{ij}\right) - \frac{1}{r}\ln\lambda_i\right] - \frac{1}{2}rVar\left[CE\left(x_i^*|\, I_{ij}\right) - \frac{1}{r}\ln\lambda_i\right] \\
&= CE\left(x_j^*\right) - \frac{1}{r}\ln\lambda_j
\end{aligned}$$

Hence, with G assumed WLOG to be connected, we have

$$\begin{aligned}
CE\left(x_i^*\right) - \frac{1}{r}\ln\lambda_i &= \frac{1}{n}\sum_{k\in N}\left(CE\left(x_k^*\right) - \frac{1}{r}\ln\lambda_k\right) \\
&= -\frac{r\sigma^2}{2n}\sum_{k\in N}\frac{1}{d_k+1} - \frac{1}{nr}\sum_{k\in N}\ln\lambda_k \quad (23)
\end{aligned}$$

On the other hand, as $x_i^* = \frac{e_i}{d_i+1} + \sum_{k\in N_i} \left(\frac{e_k}{d_k+1} - \mu_{ik}^* \right)$,

$$CE\left(x_i^*\right) = -\sum_{k\in N_i} \mu_{ik}^* - \frac{1}{2} r\sigma^2 \sum_{k\in \overline{N}_i} \frac{1}{\left(d_k+1\right)^2} \tag{24}$$

Equating the expressions for $CE\left(x_i^*\right)$ in (23) and (24), we obtain

$$\sum_{k\in N_i} \mu_{ik}^* = \frac{1}{2} r\sigma^2 \left(\frac{1}{n} \sum_{k\in N} \frac{1}{d_k+1} - \sum_{k\in \overline{N}_i} \frac{1}{\left(d_k+1\right)^2} \right) + \frac{1}{r} \left(\frac{1}{n} \sum_{k\in N} \ln \lambda_k - \ln \lambda_i \right). \tag{25}$$

Lemma 6. *Given any real vector $c \in \mathbb{R}^n$ such that $\sum_{i\in N} c_i = 0$, there exists a real vector $\mu \in \mathbb{R}^{\sum_i d_i}$ such that $\mu_{ik} + \mu_{ki} = 0$ for every linked pair ik and*

$$\sum_{k\in N_i} \mu_{ik} = c_i.$$

The solution is unique if and only if the network is minimally connected.

Lemma 6 has established that there indeed exists a solution μ^* to (25). Given any solution μ^* to (25), as $\overline{N}_i \backslash \left(N_i \backslash \overline{N}_j\right) = \overline{N}_{ij}$, we have

$$\begin{aligned}
&\sum_{k\in N_i} \mu_{ik}^* + \frac{1}{2} r\sigma^2 \sum_{k\in N_i \backslash \overline{N}_j} \frac{1}{\left(d_k+1\right)^2} + \frac{1}{r} \ln \lambda_i \\
=&\frac{1}{2} r\sigma^2 \left(\frac{1}{n} \sum_{k\in N} \frac{1}{d_k+1} - \sum_{k\in \overline{N}_{ij}} \frac{1}{\left(d_k+1\right)^2} \right) + \frac{1}{nr} \sum_{k\in N} \ln \lambda_k \\
=&\sum_{k\in N_j} \mu_{jk}^* + \frac{1}{2} r\sigma^2 \sum_{k\in N_j \backslash \overline{N}_i} \frac{1}{\left(d_k+1\right)^2} + \frac{1}{r} \ln \lambda_j
\end{aligned}$$

implying that μ^* also solves the system of equations (22). Hence, t^* is Pareto efficient. ∎

B.5 Preparatory Derivations for Proposition 4

As previewed in Section 3.3.2, we now explain in more details two preparatory steps for our main result, Proposition 4, which characterizes the Pareto efficient transfer shares under CARA-Normal setting with correlation parameter ρ.

First, we show that the Pareto efficient profile of linear and strictly bilateral transfer rules: $t_{ij}\left(e_i, e_j\right) := \alpha_{ij} e_i - \alpha_{ji} e_j + \mu_{ij}$ correspond to the solution of a complicated system of linear equations.

Lemma 7. *If there exist a vector γ such that (α, γ) jointly solve the following system of linear equations,*

$$\begin{cases} \alpha_{ij} = \frac{1}{2}\left(1 - \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} + \gamma_{ij}\right) & (26.1) \\ 0 = \alpha_{ki} - \alpha_{kj} + \gamma_{ij} \quad \forall k \in N_{ij} & (26.2) \\ \gamma_{ij} = \frac{\rho}{1+(d_{ij}+1)\rho}\left(\sum_{k \in N_i \setminus \overline{N}_j} \alpha_{ki} - \sum_{k \in N_j \setminus \overline{N}_i} \alpha_{kj}\right) & (26.3) \end{cases} \forall i, j \ s.t. \ G_{ij} = 1 \tag{26}$$

then, given any constant vector c with $c_{ij} = c_{ji}$ for all $ij \in G$, the profile of linear and strictly bilateral transfer rules defined by

$$t_{ij}(e_i, e_j) := \alpha_{ij} e_i - \alpha_{ji} e_j + c_{ij}, \quad \forall ij \in G$$

for all $ij \in G$ are Pareto efficient in $\mathcal{T}$.

We next show that instead of solving the set of linear equations (26) that imply Pareto efficiency in $\mathcal{T}$, we may solve an alternative optimization problem (11) that minimizes total consumption variances among all linear transfer rules.

Lemma 8. *$\forall \rho \in \left(-\frac{1}{n-1}, 1\right)$, if system (12) admits a unique solution, then the solution also solves system (26): i.e., a profile of linear and strictly bilateral transfer rules is Pareto efficient in $\mathcal{T}$ if it uniquely minimizes the sum of consumption variances among all profiles of linear and strictly bilateral transfer rules in $\mathcal{T}^*$.*

Finally, we show in Proposition 4 in the main text that, for any given network, system (12) indeed admits a unique solution that can be expressed in closed form. The solution depends on the pairwise correlation ρ and on the positions of individuals in the network, and can be represented as a linear function of accumulated paths along the network.

B.6 Proof of Proposition 4

Proof. Let $\overline{G} := G + \mathbf{I}_n$ so that $\overline{G}_{ii} = 1 \ \forall i \in N$. The optimality conditions given in equation (12.1) and (12.2) can be rewritten as

$$\alpha_{ji} = \overline{G}_{ij}\left(\Lambda_j - \frac{\rho}{1-\rho}\sum_{k \in N} \overline{G}_{ik}\alpha_{ki}\right) \tag{27}$$

Let $\overline{\alpha}_i := (\alpha_{1i}, \alpha_{2i}, \ldots, \alpha_{ni})'$ denote the vector of i's inflow shares, $\mathbf{\Lambda} = (\Lambda_1, \Lambda_2, \ldots, \Lambda_n)'$ the vector of rescaled constraint multipliers, and g_i represent the i-th column of $\overline{G}$. Then (27) can be rewritten in vector form as

$$\left(\mathbf{I} + \frac{\rho}{1-\rho} g_i g_i^{'}\right) \alpha_i = diag\left(g_i\right) \mathbf{\Lambda}$$

where $diag\left(g_i\right)$ is a diagonal matrix with g_i's entries on the diagonal. Left-multiplying both sides by $\left(\mathbf{I} - \frac{\rho}{1+\rho d_i} g_i g_i^{'}\right)$, which is well-defined for any $\rho > -\frac{1}{n-1}$ and any G, we have

$$\alpha_i = \left(\mathbf{I} - \frac{\rho}{1+\rho d_i} g_i g_i^{'}\right) diag\left(g_i\right) \mathbf{\Lambda}$$

As $g_i g_i^{'} \cdot diag\left(g_i\right) = g_i g_i^{'}$, the above becomes

$$\alpha_i = \left(diag\left(g_i\right) - \frac{\rho}{1+\rho d_i} g_i g_i^{'}\right) \mathbf{\Lambda} \tag{28}$$

Now, notice that (12.3) implies

$$1 = \sum_{j \in N} \alpha_{ij} = \left(d_i + 1\right) \Lambda_i - \sum_{j \in N} \overline{G}_{ij} \left(\frac{\rho}{1+\rho d_j} \sum_{k} \overline{G}_{jk} \Lambda_k\right) \tag{29}$$

and thus we have

$$\Lambda_i = \frac{1}{d_i + 1} \left(1 + \sum_{j \in \overline{N}_i} \sum_{k \in \overline{N}_j} \frac{\rho}{1+\rho d_j} \Lambda_k\right).$$

This establishes the recursive representation of the solution.

To obtain the closed-form solution, rewrite equation (29) as

$$\mathbf{1} = \sum_{i \in N} \left(diag\left(g_i\right) - \frac{\rho}{1+\rho d_i} g_i g_i^{'}\right) \mathbf{\Lambda} = \left(\overline{D} - \overline{G} \Psi \overline{G}\right) \Lambda$$

where $\overline{D}$ is a diagonal matrix with its i-th diagonal entry being $d_i + 1$, and Ψ is a diagonal matrix with its i-th diagonal entry being $\frac{\rho}{1+\rho d_i}$. Notice that $\forall \xi \in \mathbb{R}^n \backslash \{\mathbf{0}\}$,

$$\xi^{'} \left(\overline{D} - \overline{G} \Psi \overline{G}\right) \xi = \sum_{i \in N} \left(d_i + 1\right) \xi_i^2 - \sum_{i \in N} \frac{\rho}{1+\rho d_i} \left(\sum_{j \in \overline{N}_i} \xi_j\right)^2$$

$$\geq \sum_{i\in N}(d_i+1)\,\xi_i^2-\sum_{i\in N}\frac{1}{1+d_i}\left(\sum_{j\in\overline{N}_i}\xi_j\right)^2$$
$$\geq \sum_{i\in N}(d_i+1)\,\xi_i^2-\sum_{i\in N}\frac{1}{1+d_i}\cdot(1+d_i)\sum_{j\in\overline{N}_i}\xi_j^2$$
$$=\sum_{i\in N}(d_i+1)\,\xi_i^2-\sum_{i\in N}(d_i+1)\,\xi_i^2$$
$$=0$$

where the equality holds if and only if $\rho=1$ and $\xi=c\cdot\mathbf{1}$ for some $c>0$. Hence, $\forall\rho\in\left(-\frac{1}{n-1},1\right)$, $\left(\overline{D}-\overline{G}\Psi\overline{G}\right)$ is positive definite and thus invertible. Hence,

$$\Lambda=\left(\overline{D}-\overline{G}\Psi\overline{G}\right)^{-1}\mathbf{1},$$
$$\alpha_i=\left(diag\,(g_i)-\frac{\rho}{1+\rho d_i}g_ig_i^{'}\right)\left(\overline{D}-\overline{G}\Psi\overline{G}\right)^{-1}\mathbf{1}.$$

Finally, we solve for the inverse matrix above as a series of powers of $\overline{G}$. Notice that

$$\left(\overline{D}-\overline{G}\Psi\overline{G}\right)^{-1}=\left(\overline{D}^{\frac{1}{2}}\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)\overline{D}^{\frac{1}{2}}\right)^{-1}=\overline{D}^{-\frac{1}{2}}\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^{-1}\overline{D}^{-\frac{1}{2}}$$

where the middle term $\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)$ is also invertible and positive definite for $\rho\in\left(-\frac{1}{n-1},1\right)$ due to the positive definiteness of $\overline{D}-\overline{G}\boldsymbol{\Psi}\overline{G}$ and the invertibility of $\overline{D}$. For $\rho\in(0,1)$, notice that $\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}$ is also positive definite, so its eigenvalues must be positive. Also, its largest eigenvalue φ_{max} must be smaller than 1. Otherwise, there exists a nonzero vector ξ such that

$$\xi^{'}\left(I-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{\frac{1}{2}}\right)\xi=(1-\varphi_{max})\,\xi^{'}\xi<0$$

contradicting the positive definiteness of $\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)$. Then, we may write

$$\left(\mathbf{I}-\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^{-1}=\mathbf{I}+\sum_{k=1}^{\infty}\left(\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^k$$

and thus

$$\left(\overline{D}-\overline{G}\Psi\overline{G}\right)^{-1}=\overline{D}^{-1}+\overline{D}^{-\frac{1}{2}}\sum_{k=1}^{\infty}\left(\overline{D}^{-\frac{1}{2}}\overline{G}\Psi\overline{GD}^{-\frac{1}{2}}\right)^k\overline{D}^{-\frac{1}{2}}$$

$$= \overline{D}^{-1} + \sum_{k=1}^{\infty} \left(\overline{D}^{-1} \overline{Q} \right)^{k} \overline{D}^{-1}$$

where $\overline{Q} := \overline{G} \Psi \overline{G}$ can be interpreted as the weighted square of the extended adjacency matrix. Consider the set of all paths of length q between i and j under G as

$$\Pi_{ij}^{q}(G) = \left\{ (i_0, i_1, i_2, \ldots i_q) \mid i_0 = i,\ i_q = j \; and \; \overline{G}_{i_n i_{n+1}} = 1 \; for \; n = 0, 1, \ldots q-1 \right\}$$

For every $\pi_{ij} \in \Pi_{ij}^{q}(G)$, let $W(\pi_{ij})$ denote the weight associated to this path. It is not difficult to see that,

$$W(\pi_{ij}) = \frac{1}{d_i + 1} \frac{\rho}{1 + \rho d_{i_1}} \frac{1}{d_{i_2} + 1} \frac{\rho}{1 + \rho d_{i_3}} \cdots \frac{1}{d_j + 1}$$

Then

$$\begin{aligned}
\Lambda_i &= \left[\left(\overline{D} - \overline{G} \Psi \overline{G} \right)^{-1} \mathbf{1} \right]_i \\
&= \left(\overline{D}^{-1} \mathbf{1} \right)_i + \left(\sum_{k=1}^{\infty} \left(\overline{D}^{-1} \overline{Q} \right)^{k} \overline{D}^{-1} \mathbf{1} \right)_i = \frac{1}{d_i + 1} + \left[\sum_{k=1}^{\infty} \left(\overline{D}^{-1} \overline{G} \Psi \overline{G} \right)^{k} \right]_i \overline{D}^{-1} \mathbf{1} \\
&= \frac{1}{d_i + 1} + \sum_{j \in N} \left[\sum_{k=1}^{\infty} \left(\overline{D}^{-1} \overline{G} \Psi \overline{G} \right)^{k} \right]_{ij} \cdot \frac{1}{d_j + 1} = \frac{1}{d_i + 1} + \sum_{j \in N} \sum_{k=1}^{\infty} \left(\overline{D}_{[i]}^{-1} \overline{G} \Psi \overline{G} \overline{D}^{-1} ... \overline{D}^{-1} \overline{G} \Psi \overline{G}_{(j)} \right) \frac{1}{d_j + 1} \\
&= \frac{1}{d_i + 1} + \sum_{j \in N} \sum_{q=1}^{\infty} \sum_{\pi_{ij} \in \Pi_{ij}^{2q}} \left(\frac{1}{d_i + 1} \cdot \frac{\rho}{1 + \rho d_{i_1}} \cdot \frac{1}{d_{i_2} + 1} \cdot ... \right) \frac{1}{d_j + 1} \\
&= \frac{1}{d_i + 1} + \sum_{q=1}^{\infty} \sum_{j \in N} \sum_{\pi_{ij} \in \Pi_{ij}^{2q}} W(\pi_{ij})
\end{aligned}$$

This concludes the proof. ∎

B.7 Proof of Proposition 5

Proof for the Dense Case ($p_n = p$)

Proof. To start with, notice that as $d_i \sim B(n-1, p)$, we have

$$\frac{1}{n} d_i = \frac{n-1}{n} \cdot \frac{1}{n-1} d_i \xrightarrow{a.s.} p$$

and

$$\frac{d_i - np}{\sqrt{np(1-p)}} = \sqrt{\frac{n-1}{n}} \cdot \frac{d_i - (n-1)p}{\sqrt{(n-1)p(1-p)}} - \frac{p}{\sqrt{np(1-p)}} \xrightarrow{d} \mathcal{N}(0, 1).$$

For each n, set $\overline{d}_n$ by

$$\begin{aligned}\overline{d}_n &:= \left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(d_j\left(G_n\right)+1\right)^2}\middle| ij\in G_n\right]\right)^{-\frac{1}{2}}-1\\ &=\left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(2+\tilde{d}_j\right)^2}\right]\right)^{-\frac{1}{2}}-1\quad\text{where }\tilde{d}_j\sim B\left(n-2,p\right)\end{aligned}$$

so that

$$\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_j\left(G_n\right)+\frac{1}{n}\right)^2}\middle| ij\in G_n\right]-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}=0.$$

Notice that

$$\frac{1}{n}\overline{d}_n=\left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}2+\frac{n-2}{n}\cdot\frac{1}{n-2}\tilde{d}_j\right)^2}\right]\right)^{-\frac{1}{2}}-\frac{1}{n}\to\left(\frac{1}{\left(0+p\right)^2}\right)^{-\frac{1}{2}}=p.$$

Now, consider

$$\begin{aligned}&Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_i\left(G_n\right)\right]\\ =&Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)-\frac{p}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2},\ d_i\left(G_n\right)-\left(n-1\right)p\right]\\ =&\mathbb{E}_n^{ER}\left[\left(Var\left(x_i\left(G_n\right)\right)-\frac{p}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\cdot\left(d_i\left(G_n\right)-\left(n-1\right)p\right)\right]\\ &-\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)-\frac{p}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]\mathbb{E}_n^{ER}\left[d_i\left(G_n\right)-\left(n-1\right)p\right]\\ =&\mathbb{E}_n^{ER}\left[\left(Var\left(x_i\left(G_n\right)\right)-\frac{p}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\cdot\left(d_i\left(G_n\right)-\left(n-1\right)p\right)\right]\\ =&\mathbb{E}_n^{ER}\left[\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)-\frac{p}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\middle| d_i\left(G_n\right)\right]\cdot\left(d_i\left(G_n\right)-\left(n-1\right)p\right)\right]\end{aligned}$$

where the second last equality follows from the fact that

$$\mathbb{E}_n^{ER}\left[d_i\left(G_n\right)-\left(n-1\right)p\right]=0.$$

Then,

$$
\begin{aligned}
&\mathbb{E}_n^{ER}\left[nVar\left(x_i\left(G_n\right)\right)\middle| d_i\left(G_n\right)\right]\\
=&\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i+\frac{1}{n}\sum_{j\in N_i}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right]\middle| d_i\left(G_n\right)\right]\\
=&\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i+\frac{1}{n}\sum_{j\in N_i}\left\{\mathbb{E}_n^{ER}\left[\frac{1}{\left(\frac{1}{n}d_j+\frac{1}{n}\right)^2}\middle| d_i\left(G_n\right)\right]-\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right\}\\
=&\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{n}d_i\\
\xrightarrow{a.s.}&\frac{1}{\left(p+0\right)^2}\cdot 0+\frac{1}{\left(p+0\right)^2}\cdot p=\frac{1}{p}
\end{aligned}
$$

where the last equality follows from the definition of $\overline{d}_n$. By appropriate centering, we now have

$$
\begin{aligned}
&\frac{1}{\sqrt{n}}\left(d_i-\left(n-1\right)p\right)\cdot\sqrt{n}\left(n\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\middle| d_i\left(G_n\right)\right]-\frac{np}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\\
=&\frac{1}{\sqrt{n}}\left(d_i-\left(n-1\right)p\right)\cdot\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{\sqrt{n}}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{\sqrt{n}}\left(d_i-np\right)\right]\\
=&\frac{1}{\sqrt{n}}p\cdot\left[\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{\sqrt{n}}{n}+\frac{1}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\frac{1}{\sqrt{n}}\left(d_i-np\right)\right]\\
&\frac{\sqrt{n}}{n}\cdot\frac{1}{\left(\frac{1}{n}d_i+\frac{1}{n}\right)^2}\cdot\frac{1}{\sqrt{n}}\left(d_i-np\right)+\frac{p\left(1-p\right)}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\cdot\left(\frac{d_i-np}{\sqrt{np\left(1-p\right)}}\right)^2\\
\xrightarrow{d}&0+0+\frac{p\left(1-p\right)}{p^2}\cdot\chi_1^2
\end{aligned}
$$

so that

$$
\begin{aligned}
&\mathbb{E}_n^{ER}\left[\frac{1}{\sqrt{n}}\left(d_i-np\right)\cdot\sqrt{n}\left(\mathbb{E}_n^{ER}\left[nVar\left(x_i\left(G_n\right)\right)\middle| d_i\left(G_n\right)\right]-\frac{p}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\right)\right]\\
\rightarrow&\mathbb{E}\left[\frac{p\left(1-p\right)}{p^2}\cdot\chi_1^2\right]=\frac{1-p}{p}.
\end{aligned}
$$

In summary, we have

$$
nCov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_i\left(G_n\right)\right]
$$

$$
\begin{aligned}
=&\mathbb{E}_n^{ER}\left[\sqrt{n}\mathbb{E}_n^{ER}\left[nVar\left(x_i\left(G_n\right)\right)-\frac{np}{\left(\frac{1}{n}\overline{d}_n+\frac{1}{n}\right)^2}\middle| d_i\left(G_n\right)\right]\cdot\frac{1}{\sqrt{n}}\left(d_i\left(G_n\right)-\left(n-1\right)p\right)\right]\\
\to&\frac{1-p}{p}>0.
\end{aligned}
$$

∎

Proof for the Sparse Case ($np_n \to \lambda > 1$)

Proof. Suppose $np_n \to \lambda > 1$. In this case it is well known that

$$
d_i\left(G_n\right)\xrightarrow{d}Poission\left(\lambda\right),
$$

i.e.,

$$
\mathbb{P}_n^{ER}\left[d_i\left(G_n\right)=k\right]\to e^{-\lambda}\frac{\lambda^k}{k!},\quad \forall k\in\mathbb{N}.
$$

Now we set $\overline{d}_n$ as, for each n,

$$
\begin{aligned}
\overline{d}_n &:= \left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(d_j\left(G_n\right)+1\right)^2}\middle| ij\in G_n\right]\right)^{-\frac{1}{2}}-1\\
&=\left(\mathbb{E}_n^{ER}\left[\frac{1}{\left(2+\tilde{d}_j\right)^2}\right]\right)^{-\frac{1}{2}}-1\quad\text{where } \tilde{d}_j\sim B\left(n-2,p_n\right).\\
&\to d_\infty := \left(\mathbb{E}\left[\frac{1}{\left(2+Poisson\left(\lambda\right)\right)^2}\right]\right)^{-\frac{1}{2}}-1
\end{aligned}
$$

Again,

$$
\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\middle| d_i\left(G_n\right)\right]=\frac{1}{\left(d_i+1\right)^2}+\frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i
$$

and thus

$$
\begin{aligned}
&Cov_n^{ER}\left[Var\left(x_i\left(G_n\right)\right),\ d_i\left(G_n\right)\right]\\
=&\mathbb{E}_n^{ER}\left[\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\middle| d_i\left(G_n\right)\right]\cdot d_i\left(G_n\right)\right]-\mathbb{E}_n^{ER}\left[Var\left(x_i\left(G_n\right)\right)\right]\cdot\mathbb{E}_n^{ER}\left[d_i\left(G_n\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathbb{E}_n^{ER}\left[\frac{d_i}{(d_i+1)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i^2\right] - \mathbb{E}_n^{ER}\left[\frac{1}{(d_i+1)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i\right]\cdot \mathbb{E}_n^{ER}[d_i]\\
&= \mathbb{E}_n^{ER}\left[\frac{d_i}{(d_i+1)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i^2\right] - \mathbb{E}_n^{ER}\left[\frac{1}{(d_i+1)^2} + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot d_i\right]\cdot \mathbb{E}_n^{ER}[d_i]\\
&= \mathbb{E}_n^{ER}\left[\frac{d_i}{(d_i+1)^2}\right] - \mathbb{E}_n^{ER}\left[\frac{1}{(d_i+1)^2}\right]\cdot \mathbb{E}_n^{ER}[d_i] + \frac{1}{\left(\overline{d}_n+1\right)^2}\cdot Var_n^{ER}[d_i]\\
&\to \kappa(\lambda) := \mathbb{E}\left[\frac{\xi-\lambda}{(\xi+1)^2}\right] + \frac{\lambda}{(d_\infty+1)^2}, \quad \text{where } \xi \sim Poisson(\lambda)\\
&= \mathbb{E}\left[\frac{\xi}{(\xi+1)^2}\right] - \lambda\mathbb{E}\left[\frac{1}{(\xi+1)^2} - \frac{1}{(\xi+2)^2}\right], \quad \text{where } \xi \sim Poisson(\lambda)\\
&= \mathbb{E}\left[\frac{\xi^3+4\xi^2+2(2-\lambda)\xi-3\lambda}{(\xi+1)^2(\xi+2)^2}\right], \quad \text{where } \xi \sim Poisson(\lambda)
\end{aligned}
$$

which may be positive or negative depending on λ. ∎

Numerical computation of $\kappa(\lambda)$ in in Mathematica shows that $\kappa(\lambda)$ is positive for large enough λ with a cutoff $\overline{\lambda} \approx 3.8803$. See Figure 3 for numerical plots of $\kappa(\lambda)$.

B.8 Proof of Proposition 6

Proof. Let $e_i := y_i - \mathbb{E}[y_i]$. The weighted local equal sharing rule can be rewritten as:

$$
t_{ij}^* = \frac{1/r_j}{\sum_{k\in\overline{N}_i} 1/r_k}\cdot e_i - \frac{1/r_i}{\sum_{k\in\overline{N}_j} 1/r_k}\cdot e_j + \mu_{ij}
$$

for some constant $\mu_i j$.

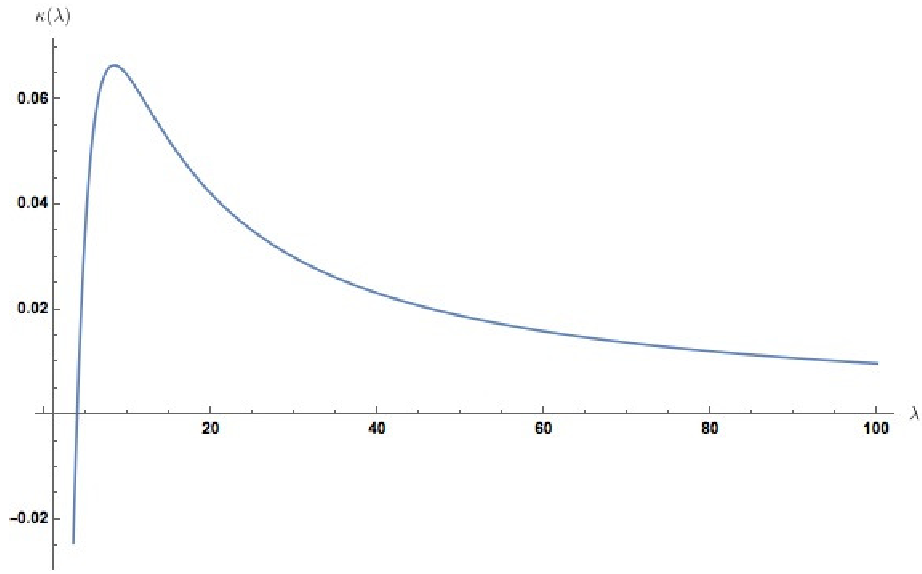
The local Borch rule for Pareto efficiency with heterogeneous risk-aversion parameters requires that $r_i CE(x_i^*|\, I_{ij}) - r_j CE\left(x_j^*\middle|\, I_{ij}\right)$ be constant across realizations of I_{ij}, or equivalently,

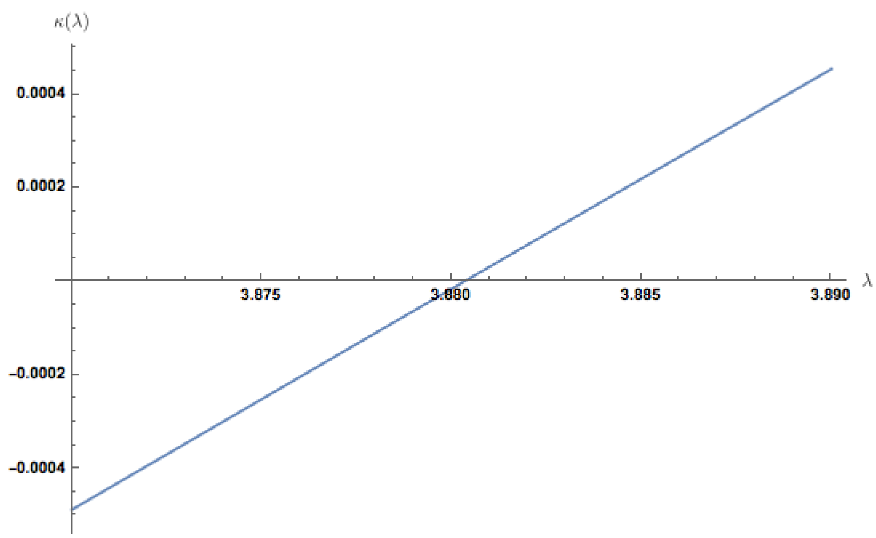
$$
r_i E[x_i^*|I_{ij}] - \frac{1}{2}r_i^2 Var(x_i^*|\, I_{ij}) = r_j E[x_j^*|I_{ij}] - \frac{1}{2}r_j^2 Var\left(x_j^*\middle|\, I_{ij}\right) + C_{ij} \qquad (30)
$$

We show that this requirement is satisfied by the weighted local equal sharing rule t^*:

$$
r_i E[x_i^*|I_{ij}] = \sum_{k\in\overline{N}_{ij}} \frac{1}{\sum_{h\in\overline{N}_h} 1/r_k} e_k - r_i \sum_{k\in\overline{N}_i} \mu_{ik}
$$

Figure 3: Plot of $\kappa(\lambda)$





and

$$r_j E[x_j^*|I_{ij}] = \sum_{k\in\overline{N}_{ij}} \frac{1}{\sum_{h\in\overline{N}_h} 1/r_k} e_k - r_j \sum_{k\in\overline{N}_j} \mu_{jk}$$

so that

$$r_i E[x_i^*|I_{ij}] - r_j E[x_j^*|I_{ij}] = \text{constant}$$

while, with $\sigma_k^2 := Var(e_k)$,

$$Var\left(x_i^*\middle| I_{ij}\right) = \sum_{k\in N_i\setminus\overline{N}_j} \left(\frac{1/r_j}{\sum_{h\in\overline{N}_h} 1/r_k}\right)^2 \sigma_k^2,$$

is also a constant, i.e., does not depend on the realization of I_{ij}, and similarly for $Var\left(x_j^*\middle| I_{ij}\right)$. Hence, equation (30) above is satisfied.

The weighted local equal sharing rule t^*above induces the following final consumption plan

$$x_i^* = \sum_{k\in\overline{N}_i} \frac{1/r_i}{\sum_{k\in\overline{N}_k} 1/r_k} e_k,$$

where it is clear that individual i's shock exposures will be smaller when individual i is more risk-averse (i.e., when r_i is larger). ∎

B.9 Detailed Specification and Proof for Proposition 7

Specifically, we assume that the correlation between e_i and e_j geometrically decays with social distance between i and j :

$$Corr\left(e_i, e_j\right) = \varrho^{dist(i,j)},$$

where the social distance $dist\left(i, j\right)$ is formally defined as the length (i.e., the number of links) of the shortest path connecting i and j in network G. For notational simplicity we set $\sigma^2 = 1$.

For tractability, we restrict attention to circle networks with $n = 2m + 1$ individuals. A n-circle consists of n individuals and n links: $G_{i,i+1} = 1$ for $i = 1, ..., n$.[46] For any linked pair $i, i+1$ along a n-circle (with $n \geq 4$), the conditional distribution of e_{i-1} (and similarly for e_{i+2}) is

$$e_{i-1}|_{e_i, e_{i+1}} \sim \mathcal{N}\left(\varrho e_i, 1 - \varrho\right).$$

Following a similar argument as in Section 4.2, we obtain the following condition for Pareto

[46] We, for notational simplicity, define individual $n+1$ to be individual 1, and individual 0 to be individual n.

efficiency subject to local information constraints:

$$\begin{cases} \alpha_{i,i+1} & = \frac{1}{2}\left(1-\alpha_{i,i-1}+\varrho\alpha_{i-1,i}\right) \\ \alpha_{i+1,i} & = \frac{1}{2}\left(1-\alpha_{i+1,i+2}+\varrho\alpha_{i+2,i+1}\right) \end{cases}$$

for all $i \in N$. Then, the unique and symmetric solution for the above system is given by

$$\alpha_{ij}^{*} \equiv \alpha^{geo}\left(\varrho\right) = \frac{1}{3-\varrho} \quad \forall G_{ij}=1.$$

Under α^*, the final consumption for each individual is

$$x_i^{geo}\left(\varrho\right) = \frac{1}{3-\varrho}e_{i-1} + \frac{1-\varrho}{3-\varrho}e_i + \frac{1}{3-\varrho}e_{i+1}$$

with a variance of

$$Var_{geo,\varrho}\left(x_i^{geo}\left(\varrho\right)\right) = \frac{1+\varrho}{3-\varrho}.$$

In comparison, under the symmetric correlation structure in Section 4.2, the condition for Pareto efficiency on a n-circle is

$$\alpha_{i,i+1} = \frac{1}{2}\left[1-\alpha_{i,i-1}+\frac{\rho}{1+\rho}\left(\alpha_{i-1,i}-\alpha_{i+2,i+1}\right)\right]$$

with its unique and symmetric solution being

$$\alpha_{ij} \equiv \alpha^{unif}\left(\rho\right) = \frac{1}{3} \quad \forall G_{ij}=1,$$

which is exactly the local equal sharing rule. This implies a final consumption of

$$x_i^{unif}\left(\rho\right) = \frac{1}{3}e_{i-1} + \frac{1}{3}e_i + \frac{1}{3}e_{i+1}$$

with a variance of

$$Var_{unif,\rho}\left(x_i^{unif}\left(\rho\right)\right) = \frac{1+2\rho}{3}.$$

We compare the correlation structures by setting ρ and ϱ to be such that each individual's consumption variance is equalized across the two correlation structures under the global equal sharing rule (which achieves first best risk sharing):

$$x_i^{FB} = \frac{1}{n}\sum_{k\in N} e_k.$$

The consumption variances that this sharing rule implies for the two correlation structures are:

$$Var_{unif,\rho}\left(x_i^{FB}\right) = \frac{1+2m\rho}{2m+1}$$
$$Var_{geo,\varrho}\left(x_i^{FB}\right) = \frac{1+2\sum_{k=1}^{m}\varrho^k}{2m+1} = \frac{2\frac{1-\varrho^{m+1}}{1-\varrho}-1}{2m+1},$$

The first-best total variances under the two correlations structures are equal if and only if

$$Var_{unif,\rho}\left(x_i^{FB}\right) = Var_{geo,\varrho}\left(x_i^{FB}\right) \Leftrightarrow \frac{1+2m\rho}{2m+1} = \frac{2\frac{1-\varrho^{m+1}}{1-\varrho}-1}{2m+1}$$
$$\Leftrightarrow \rho = \rho_m\left(\varrho\right) := \frac{\varrho\left(1-\varrho^m\right)}{m\left(1-\varrho\right)}.$$

Noticing that the total variances without risk sharing at all are both equal to $(2m+1)$ under either correlation structure, setting $\rho = \rho_m\left(\varrho\right)$ implies that the total amount of shareable risk is equalized between the two correlation structures. Next we compare the consumption variances given Pareto efficient risk-sharing arrangements subject to local information constraints.

Notice that

$$Var_{unif,\rho}\left(x_i^{unif}\left(\rho\right)\right) \leq Var_{geo,\varrho}\left(x_i^{geo}\left(\varrho\right)\right) \quad \Leftrightarrow \quad \rho \leq \overline{\rho}\left(\varrho\right) := \frac{2\varrho}{3-\varrho}.$$

Hence, whenever

$$m > \frac{\left(3-\varrho\right)\left(1-\varrho^m\right)}{2\left(1-\varrho\right)}$$

we will have $\rho\left(\varrho\right) < \overline{\rho}\left(\varrho\right)$ and thus $Var_{uni}^{\rho}\left(x_i^{unif}\left(\rho\right)\right) < Var_{geo}^{\varrho}\left(x_i^{geo}\left(\varrho\right)\right)$. In other words, fixing ϱ, efficient risk sharing subject to the local information constraint performs strictly better under the uniform correlation setting than under the geometrically decaying setting.

Moreover, the difference can be very stark. As $m \to \infty$,

$$\rho = \rho\left(\varrho\right) = \frac{\varrho\left(1-\varrho^m\right)}{m\left(1-\varrho\right)} \to 0,$$

and thus

$$Var_{unif,\rho}\left(x_i^{unif}\left(\rho\right)\right) = \frac{1+2\rho}{3} \to \frac{1}{3}, \quad \text{as } m \to \infty$$

while

$$Var_{geo,\rho}\left(x_i^{geo}\left(\varrho\right)\right) = \frac{1+\varrho}{3-\varrho} \quad \forall m.$$

When also taking $\varrho \to 1$ (after taking $m \to \infty$), we get

$$\lim_{\varrho \to 1} \lim_{m \to \infty} Var_{unif,\rho(\varrho)} \left(x_i^{unif} \left(\rho \left(\varrho \right) \right) \right) = \frac{1}{3},$$
$$\lim_{\varrho \to 1} \lim_{m \to \infty} Var_{geo,\rho} \left(x_i^{geo} \left(\varrho \right) \right) = 1.$$

B.10 Quadratic Utility Functions

With quadratic utility functions $u_i(x_i) = x_i - \frac{1}{2} r x_i^2$, the localized Borch rule requires that

$$\frac{\lambda_j}{\lambda_i} = \frac{\mathbb{E}_{ij}\left[u_i'(x_i)\right]}{\mathbb{E}_{ij}\left[u_i'(x_j)\right]} = \frac{\mathbb{E}_{ij}\left[1 - r x_i\right]}{\mathbb{E}_{ij}\left[1 - r x_j\right]}$$

$$\begin{aligned}
\Leftrightarrow \quad & \lambda_i - \lambda_i r \left(\mu_i + e_i - t_{ij} - \sum_{h \in N_i} E_{ij}\left[t_{ih}\right] \right) = \lambda_j - \lambda_j \left(\mu_j + e_j + t_{ij} - \sum_{h \in N_j} \mathbb{E}_{ij}\left[t_{jh}\right] \right) \\
\Leftrightarrow \quad & r\left(\lambda_i + \lambda_j\right) t_{ij} = -\left(\lambda_i - \lambda_j\right) + \lambda_i r \left(\mu_i + e_i - \sum_{h \in N_i \backslash j} \mathbb{E}_{ij}\left[t_{ih}\left(I_{ih}\right)\right] \right) \\
& \qquad - \lambda_j r \left(e_j - \sum_{h \in N_j \backslash i} \mathbb{E}_{ij}\left[t_{jh}\left(I_{jh}\right)\right] \right) \\
\Leftrightarrow \quad & t_{ij} = \frac{\lambda_i}{\lambda_i + \lambda_j} \left(\mu_i + e_i - \sum_{h \in N_i \backslash j} \mathbb{E}_{ij}\left[t_{ih}\left(I_{ih}\right)\right] \right) \\
& \qquad - \frac{\lambda_j}{\lambda_i + \lambda_j} \left(\mu_j + e_j - \sum_{h \in N_j \backslash i} \mathbb{E}_{ij}\left[t_{jh}\left(I_{jh}\right)\right] \right) - \frac{\lambda_i - \lambda_j}{r\left(\lambda_i + \lambda_j\right)}
\end{aligned}$$

Postulating a bilateral linear rule:

$$t_{ij}\left(I_{ij}\right) = \alpha_{ij} e_i - \alpha_{ji} e_j + c_{ij}$$

Notice that this is equivalent to specifying $t_{ij}\left(I_{ij}\right) = \alpha_{ij} y_i - \alpha_{ji} y_j + c_{ij}$ as we allow μ_{ij} are simultaneously determined along with:

$$x_i = \left(1 - \sum_{j \in N_i} \alpha_{ij} \right) e_i + \sum_{j \in N_i} \alpha_{ji} e_j + \mu_i - \sum_{j \in N_i} c_{ij}$$

$$\equiv \left(1-\sum_{j\in N_i}\alpha_{ij}\right)y_i+\sum_{j\in N_i}\alpha_{ji}y_j+\left(\sum_{j\in N_i}\alpha_{ji}\mu_i-\sum_{j\in N_i}\alpha_{ij}\mu_j\right)-\sum_{j\in N_i}c_{ij}$$

Plugging in the postulation,

$$\begin{aligned}
t_{ij} = {} & \frac{\lambda_i}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_i\setminus j}\alpha_{ih}\right)e_i+\sum_{h\in N_{ij}}\alpha_{hi}e_h+\sum_{h\in N_i\setminus\overline{N}_j}\alpha_{hi}\mathbb{E}_{ij}\left[e_h\right]\right)\\
& -\frac{\lambda_j}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_j\setminus i}\alpha_{jh}\right)e_j+\sum_{h\in N_{ij}}\alpha_{hj}e_h+\sum_{h\in N_j\setminus\overline{N}_i}\alpha_{hi}\mathbb{E}_{ij}\left[e_h\right]\right)\\
& +\frac{\lambda_i\left(\mu_i-\sum_{h\in N_i\setminus j}c_{ih}\right)-\lambda_j\left(\mu_j-\sum_{h\in N_j\setminus i}c_{jh}\right)}{\lambda_i+\lambda_j}-\frac{\lambda_i-\lambda_j}{r\left(\lambda_i+\lambda_j\right)}\\
= {} & \frac{\lambda_i}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_i\setminus j}\alpha_{ih}\right)e_i+\sum_{h\in N_{ij}}\alpha_{hi}e_h+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\sum_{h\in N_i\setminus\overline{N}_j}\alpha_{hi}\cdot\sum_{k\in\overline{N}_{ij}}e_k\right)\\
& -\frac{\lambda_j}{\lambda_i+\lambda_j}\left(\left(1-\sum_{h\in N_j\setminus i}\alpha_{jh}\right)e_j+\sum_{h\in N_{ij}}\alpha_{hj}e_h+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\sum_{h\in N_j\setminus\overline{N}_i}\alpha_{hi}\cdot\sum_{k\in\overline{N}_{ij}}e_k\right)\\
& +\frac{\lambda_i\left(\mu_i-\sum_{h\in N_i\setminus j}c_{ih}\right)-\lambda_j\left(\mu_j-\sum_{h\in N_j\setminus i}c_{jh}\right)}{\lambda_i+\lambda_j}-\frac{\lambda_i-\lambda_j}{r\left(\lambda_i+\lambda_j\right)}
\end{aligned}$$

given that

$$\mathbb{E}_{ij}[e_k]=\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\sum_{k\in\overline{N}_{ij}}e_k$$

whenever the joint distribution of endowments e belong to the elliptical family of distributions.

In the special case of equal weighting: $\lambda_i=\lambda_j$, we have

$$\begin{aligned}
\alpha_{ij} &= \frac{1}{2}\left(1-\sum_{h\in N_i}\alpha_{ih}+\frac{\rho}{1+\left(d_{ij}+1\right)\rho}\left(\sum_{h\in N_i\setminus\overline{N}_j}\alpha_{hi}-\sum_{h\in N_j\setminus\overline{N}_i}\alpha_{hi}\right)\right)\\
c_{ij} &= \frac{1}{2}\left(\mu_i-\mu_j\right)-\frac{1}{2}\left(\sum_{h\in N_i\setminus j}c_{ih}-\sum_{h\in N_j\setminus i}c_{jh}\right)
\end{aligned}$$

Note that system of linear equations in α is exactly the same one as in Section 4.

B.11 Approximate Pareto Efficiency of the Local Equal Sharing Rule in Star Networks with CRRA Utilities

B.11.1 Analytical

Next, we illustrate that the local equal sharing rule remains approximately Pareto efficient in an asymmetric star network under CRRA utility function.

Formally, suppose that G is a star network with N individuals. Let $u_i(x) := \log(x)$ be the common CRRA utility function, and the endowments be i.i.d. with an illustrative uniform distribution, $y_i \sim_{i.i.d.} Uniform\left(\underline{y}, \underline{y}+1\right)$ for some $\underline{y} > 0$. Write $\mu := \mathbb{E}[y_i] = \underline{y} + 0.5$ and $e_i := y_i - \mu \sim_{i.i.d.} Uniform(-0.5, 0.5)$.

We again focus on linear transfer rules of the form $t_{0j} = \alpha_c e_0 - \alpha_p e_j - \beta$ where 0 denotes the center individual, $j = 1, ..., N-1$ denotes a peripheral individual and t_{0j} denotes the net transfer from the center to a generic peripheral individual. The final consumption implied by the transfer rule above is then given by:

$$x_0 = \mu + (N-1)\beta + (1 - (N-1)\alpha_c) e_0 + \alpha_p \sum_{j=1}^{N-1} e_j$$

$$x_j = \mu - \beta + \alpha_c e_0 + (1 - \alpha_p) e_j$$

3-Star

When $N = 3$, we have

$$x_0 = \mu + 2\beta + (1 - 2\alpha_c) e_0 + \alpha_p e_j + \alpha_p e_k$$
$$x_j = \mu - \beta + \alpha_c e_0 + (1 - \alpha_p) e_j$$

with

$$\begin{aligned}
\mathbb{E}\left[u'(x_0) \middle| e_0, e_j\right] &= \mathbb{E}\left[\frac{1}{\mu + 2\beta + (1-2\alpha_c) e_0 + \alpha_p e_j + \alpha_p e_k} \middle| e_0, e_j\right] \\
&= \int \frac{1}{\mu + 2\beta + (1-2\alpha_c) e_0 + \alpha_p e_j + \alpha_p e_k} de_k \\
&= \frac{1}{\alpha_p} \log\left(\mu + 2\beta + (1-2\alpha_c) e_0 + \alpha_p e_j + \alpha_p e_k\right)\Big|_{-0.5}^{0.5} \\
&= \frac{1}{\alpha_p} \log\left(\frac{\mu + 2\beta + (1-2\alpha_c) e_0 + \alpha_p e_j + 0.5\alpha_p}{\mu + 2\beta + (1-2\alpha_c) e_0 + \alpha_p e_j - 0.5\alpha_p}\right)
\end{aligned}$$

and

$$\mathbb{E}\left[u^{'}\left(x_j\right)\Big| e_0, e_j\right] = \frac{1}{\mu - \beta + \alpha_c e_0 + \left(1 - \alpha_p\right) e_j}.$$

For this linear transfer rule t_{0j} to be optimal, by the local Borch rule we need

$$\log\left(1 + \frac{\alpha_p}{\mu + 2\beta + \left(1 - 2\alpha_c\right) e_0 + \alpha_p e_j - 0.5\alpha_p}\right) = \frac{\alpha_p}{\mu - \beta + \alpha_c e_0 + \left(1 - \alpha_p\right) e_j}$$

which cannot be satisifed for any choice of $(\alpha_c, \alpha_p, \beta)$.

However, we now show that the local equal sharing rule with $\alpha_c = \frac{1}{3}$ and $\alpha_p = \frac{1}{2}$ is approximately Pareto efficient. Specifically, we utilize the well-known inequality

$$\frac{x}{1+x} \leq \log\left(1+x\right) \leq x$$

as the basis of our approximation. Notice that this approximation should be very good when $\mu = \underline{y} + 0.5$ is very large.

Based on the upper bound $\log\left(1+x\right) \leq x$, the local Borch rule requires

$$\frac{\alpha_p}{\mu - \beta + \alpha_c e_0 + \left(1 - \alpha_p\right) e_j} \leq \frac{\alpha_p}{\mu + 2\beta + \left(1 - 2\alpha_c\right) e_0 + \alpha_p e_j - 0.5\alpha_p}$$

Based on the lower bound $\frac{x}{1+x} \leq \log\left(1+x\right)$, the local Borch rule requires

$$\begin{aligned}\frac{\frac{\alpha_p}{\mu+2\beta+(1-2\alpha_c)e_0+\alpha_p e_j-0.5\alpha_p}}{1+\frac{\alpha_p}{\mu+2\beta+(1-2\alpha_c)e_0+\alpha_p e_j-0.5\alpha_p}} &= \frac{\alpha_p}{\mu + 2\beta + \left(1 - 2\alpha_c\right) e_0 + \alpha_p e_j + 0.5\alpha_p} \\ &\leq \frac{\alpha_p}{\mu - \beta + \alpha_c e_0 + \left(1 - \alpha_p\right) e_j}\end{aligned}$$

To satisfy the two inequalities above for all e_0, e_j, we can set $\alpha_c = \frac{1}{3}, \quad \alpha_p = \frac{1}{2}$, and β to be any number such that $2\beta - \frac{1}{4} \leq -\beta \leq 2\beta + \frac{1}{4}$, i.e.

$$\beta \in \left[-\frac{1}{12}, \frac{1}{12}\right].$$

This suggests that the local equal sharing rule $t_{0j}^{*} = \frac{1}{3}e_0 - \frac{1}{2}e_j$ is approximately Pareto efficient.

N-Star

The argument above can be inductively generalized to N- star so that any local equal sharing rule of the form $t_{0j}^{*} = \frac{1}{3}e_0 - \frac{1}{2}e_j + \beta$ with

$$\beta \in \left[-\frac{N-2}{4N}, \frac{N-2}{4N}\right] \to \left[-\frac{1}{4}, \frac{1}{4}\right]$$

approximately satisfies the local Borch rule based on $\frac{x}{1+x} \leq \log(1+x) \leq x$.

B.11.2 Numerical

We now numerically illustrate the approximate optimality of the local equal sharing rule in star networks with CRRA utility functions under the following configurations:

Networks are configured to be star with $N = 3, 4, 5$ individuals.

Utility functions are taken to be CRRA $u(x) = \frac{1}{1-\theta}(5+x)^{1-\theta}$ with parameter $\theta = 0.5, 1, 2$, where $\theta = 1$ corresponds to the log utility function $u(x) = \log(x)$.

Log-Normal Endowments

Endowments are i.i.d. drawn from the standard log-normal distribution: $y_i \sim_{iid} \text{lognormal}(0, 1)$.

We then numerically solve for the best linear transfer shares among symmetric linear transfer rules of the form $t_{0j} := \alpha_c e_0 - \alpha_p e_j - \beta$, with "0" denoting the center individual, "j" denoting a peripheral individual, and $e_i := y_i - \mathbb{E}[y_i]$ be the net endowment shock.

Note that the local equal sharing rule corresponds to $\alpha_c^* = \frac{1}{N}$ and $\alpha_p^* = \frac{1}{2}$ under this setting. The following table we report the numerical solutions for the optimal $(\alpha_c, \alpha_p, \beta)$, which are reasonably close to α_c^* and α_p^* . Moreover, the magnitudes of β are relatively small, which is also consistent with the analytical approximation result we obtained above.

θ	N	α_c	α_p	β
0.5	3	0.340	0.48	0.02
0.5	4	0.253	0.48	0.04
0.5	5	0.208	0.49	0.04
1	3	0.340	0.49	0.03
1	4	0.253	0.49	0.05
1	5	0.208	0.49	0.05
2	3	0.340	0.49	0.04
2	4	0.253	0.49	0.06
2	5	0.205	0.48	0.06

Bernoulli Endowments

Endowments are i.i.d. drawn from as $y_i \sim_{iid} 1 + Bernoulli\left(\frac{1}{2}\right)$.

With binary distribution of endowments, we can completely characterizing the (symmetric) transfer rules with four parameters without imposing linearity as a restriction:

$$t_{0j}(y_0, y_j) = \begin{cases} \alpha_{22}, & \text{if } y_0 = y_j = 2 \\ \alpha_{12} & \text{if } y_0 = 1, y_j = 2 \\ \alpha_{21}, & \text{if } y_0 = 2, y_j = 1 \\ \alpha_{11} & \text{if } y_0 = y_j = 1 \end{cases}$$

Writing $e_i = y_i - \mathbb{E}[y_i] = y_i - 1.5$, the local equal sharing rule $t^*_{0j} = \frac{1}{N}e_0 - \frac{1}{2}e_j$ corresponds to the following restriction on α:

$$\alpha^*_{22} - \alpha^*_{12} = \alpha^*_{21} - \alpha^*_{11} = \frac{1}{N}$$
$$\alpha^*_{22} - \alpha^*_{21} = \alpha^*_{12} - \alpha^*_{11} = \frac{1}{2}$$

The following table reports the numerical solutions we obtained, which again confirm the optimality of the local equal sharing rule.

θ	N	α_{11}	α_{12}	α_{21}	α_{22}	$\alpha_{22} - \alpha_{12}$	$\alpha_{22} - \alpha_{21}$
1	3	0.083	-0.417	0.417	-0.083	0.335	0.5
1	4	0.125	-0.375	0.375	-0.125	0.25	0.5
1	5	0.15	-0.35	0.35	-0.15	0.2	0.5

C Additional Proofs and Supporting Materials

C.1 Proofs of Lemmas 1-5

Lemma. 1: *$\mathcal{T}^*$ with $\langle \cdot, \cdot \rangle$ forms an inner product space.*

Proof. We first show that $\langle \cdot, \cdot \rangle$ is a well-defined inner product. Symmetry immediately follows from the definition. Linearity in the first argument follows from the linearity of the expectation operator:

$$\begin{aligned} \langle \alpha s + \beta t,\ r \rangle &= \mathbb{E}\left[\sum_{G_{ij}=1} (\alpha s_{ij} + \beta t_{ij})\, r_{ij}\right] = \alpha \mathbb{E}\left[\sum_{G_{ij}=1} s_{ij} r_{ij}\right] + \beta \mathbb{E}\left[\sum_{G_{ij}=1} t_{ij} r_{ij}\right] \\ &= \alpha \langle s,\ r \rangle + \beta \langle t,\ r \rangle . \end{aligned}$$

Positive definiteness is also obvious: $\langle t, t \rangle = \mathbb{E}\left[\sum_{G_{ij}=1} t_{ij}^2 (e)\right] \geq 0$ and $\langle t, t \rangle = 0$ if and only if $t = \mathbf{0}$, i.e., $t_{ij}(\omega) = 0$ for all linked ij and $\mathbb{P}$-almost all $e \in \Omega$.

We then show that $\mathcal{T}$ is a linear space. $\forall s, t \in \mathcal{T}$, $\forall \alpha, \beta \in \mathbb{R}$, $\alpha s(I_{ij}) + \beta t(I_{ij})$ is also $\sigma(I_{ij})$-measurable, and

$$\alpha s_{ij}(e) + \beta t_{ij}(e) = -\left(\alpha s_{ji}(e) + \beta t_{ji}(e)\right).$$

Finiteness of expectation is obvious. Hence, $\alpha s + \beta t \in \mathcal{T}$. ∎

Lemma. 2: *The objective function in (3)*

$$J(t) := \mathbb{E}\left[\sum_{k \in N} \lambda_k u_k \left(e_k - \sum_{h \in N_k} t_{kh}(e)\right)\right]$$

is concave on $\mathcal{T}$.

Proof. $\forall s, t \in \mathcal{T}$, $\forall \alpha \in [0,1]$,

$$
\begin{aligned}
& J\left(\alpha s+(1-\alpha) t\right) \\
& =\mathbb{E}\left[\sum_i \lambda_i u_i\left(e_i-\sum_{j \in N_i}\left(\alpha s_{ij}\left(e\right)+(1-\alpha) t_{ij}\left(e\right)\right)\right)\right] \\
& =\sum_i \lambda_i \mathbb{E}\left[u_i\left(\alpha\left(e_i-\sum_{j \in N_i} s_{ij}\left(e\right)\right)+(1-\alpha)\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)\right)\right)\right] \\
& \geq \sum_i \lambda_i \mathbb{E}\left[\alpha u_i\left(e_i-\sum_{j \in N_i} s_{ij}\left(e\right)\right)+(1-\alpha) u_i\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)\right)\right] \\
& =\alpha \mathbb{E}\left[\sum_i \lambda_i u_i\left(e_i-\sum_{j \in N_i} s_{ij}\left(e\right)\right)\right]+(1-\alpha) \mathbb{E}\left[\sum_i \lambda_i u_i\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)\right)\right] \\
& =\alpha J\left(s\right)+(1-\alpha) J\left(t\right).
\end{aligned}
$$

∎

Lemma. 3: *J is Gâteaux-differentiable.*

Proof. $\forall s, t \in \mathcal{T}$, for $\alpha > 0$,

$$
\begin{aligned}
& \frac{J\left(t+\alpha s\right)-J\left(t\right)}{\alpha} \\
& =\mathbb{E}\left[\sum_i \lambda_i\left[\frac{u_i\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)-\alpha \sum_{j \in N_i} s_{ij}\left(e\right)\right)-u_i\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)\right)}{\alpha}\right]\right] \\
& =\mathbb{E}\left[\sum_i \lambda_i\left[-\frac{u_i^{'}\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)-\alpha \tilde{s}\left(e\right)\right) \cdot \alpha \sum_{j \in N_i} s_{ij}\left(e\right)}{\alpha}\right]\right] \\
& \qquad \text{for some } \tilde{s}_{ij}(e) \text{ between } 0 \text{ and } \sum_{j \in N_i} s_{ij}\left(e\right) \\
& =-\mathbb{E}\left[\sum_i \lambda_i\left[u_i^{'}\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)-\alpha \tilde{s}\right) \cdot \sum_{j \in N_i} s_{ij}\left(e\right)\right]\right] \\
& \rightarrow-\mathbb{E}\left[\sum_i \lambda_i\left[u_i^{'}\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)\right) \cdot \sum_{j \in N_i} s_{ij}\left(e\right)\right]\right] \text{ as } \alpha \rightarrow 0 \\
& =-\sum_i \lambda_i \mathbb{E}\left[u_i^{'}\left(e_i-\sum_{j \in N_i} t_{ij}\left(e\right)\right) \mathbf{1}_{i \times N_i} \cdot s\left(e\right)\right] \\
& =\sum_i \lambda_i<f_i, s>
\end{aligned}
$$

where

$$f_i(e) := -u_i^{'}\left(e_i - \sum_{j \in N_i} t_{ij}(e)\right)\mathbf{1}_{i \times N_i}$$

and $\mathbf{1}_{i \times N_i}$ is vector of 0 and 1s that equals 1 for the (directed) link ij for any $j \in N_i$ so that $\mathbf{1}_{i \times N_i} \cdot s(e) = \sum_{j \in N_i} s_{ij}(e)$. Define $J^{'}(t) : \mathcal{T} \to \mathbb{R}$ by

$$J^{'}(t)\, s = \sum_i \lambda_i < f_i, s > .$$

Clearly $J^{'}(t)$ is a linear operator on $\mathcal{T}$, and is thus the Gâteaux-derivative of J. ∎

Lemma. 4: *For any $t \in \mathcal{T}$ that solves (4), we have*

$$J^{'}(t) = \mathbf{0}.$$

Proof. To solve (4)

$$\max_{\tilde{t}_{ij} \in \mathbb{R}} J^{(ij, I_{ij})}\left(\tilde{t}_{ij}\right) := \mathbb{E}\left[\lambda_i u_i\left(e_i - \tilde{t}_{ij} - \sum_{h \in N_i} t_{ih}\right) + \lambda_j u_j\left(e_j + \tilde{t}_{ij} - \sum_{h \in j} t_{jh}\right)\middle| I_{ij}\right]$$

we first notice the objective function $J^{(ij,I_{ij})}\left(\tilde{t}_{ij}\right)$ is strictly concave in $\tilde{t}_{ij}$ on $\mathbb{R}$. Hence, the sufficient and necessary condition for optimality is given by the FOC:

$$\mathbb{E}\left[\lambda_i u_i^{'}\left(e_i - \sum_{h \in N_i} t_{ih}(e)\right)\middle| I_{ij}\right] = \mathbb{E}\left[\lambda_j u_j^{'}\left(e_j - \sum_{h \in N_j} t_{jh}(e)\right)\middle| I_{ij}\right]$$

Then, $\forall s \in \mathcal{T}$,

$$\begin{aligned}
J^{'}(t)\, s &= -\mathbb{E}\left[\sum_i \lambda_i \left[u_i^{'}\left(e_i - \sum_{j \in N_i} t_{ij}(e)\right) \cdot \sum_{j \in N_i} s_{ij}(e)\right]\right] \\
&= -\frac{1}{2}\sum_{G_{ij}=1} \mathbb{E}\left[\left(\lambda_i u_i^{'}\left(e_i - \sum_{h \in N_i} t_{ih}(e)\right) - \lambda_j u_j^{'}\left(e_j - \sum_{h \in N_j} t_{jh}(e)\right)\right) \cdot s_{ij}(e)\right] \\
&= -\frac{1}{2}\sum_i \sum_{j \in N_i} \mathbb{E}\left[s_{ij}(I_{ij}) \cdot \mathbb{E}\left[\lambda_i u_i^{'}\left(e_i - \sum_{h \in N_i} t_{ih}(e)\right) - \lambda_j u_j^{'}\left(e_j - \sum_{h \in N_j} t_{jh}(e)\right)\middle| I_{ij}\right]\right] \\
&= -\frac{1}{2}\sum_i \sum_{j \in N_i} \mathbb{E}\left[s_{ij}(I_{ij}) \cdot 0\right]
\end{aligned}$$

$= 0.$

Hence $J^{'}(t) = \mathbf{0}$. ∎

Lemma. 5:*The set of consumption plan induced by the profiles of transfer rules t in $\mathcal{T}$ is convex.*

Proof. Let $x, x^{'}$ be two profiles of consumption plans induced by $t, t^{'}$ respectively. Then $\forall \lambda \in [0,1]$,

$$\begin{aligned}\lambda x_i(e) + (1-\lambda) x_i^{'}(e) &= \lambda \left[e_i - \sum_{j \in N_i} t_{ij}(e)\right] + (1-\lambda)\left[e_i - \sum_{j \in N_i} t_{ij}^{'}(e)\right] \\ &= e_i - \sum_{j \in N_u} \left[\lambda t_{ij}(e) + (1-\lambda) t_{ij}^{'}(e)\right]\end{aligned}$$

Thus $\left(\lambda x + (1-\lambda) x^{'}\right)$ can be induced by $\left(\lambda t + (1-\lambda) t^{'}\right)$. $\mathcal{T}$, as an inner product space, is convex, so the set of consumption plans induced by the profiles of transfer rules in $\mathcal{T}$ must also be convex. ∎

C.2 Proof of Lemma 6

Lemma. 6: *Given any real vector $c \in \mathbb{R}^n$ such that $\sum_{i \in N} c_i = 0$, there exists a real vector $\mu \in \mathbb{R}^{\sum_i d_i}$ such that $\mu_{ik} + \mu_{ki} = 0$ for every linked pair ik and*

$$\sum_{k \in N_i} \mu_{ik} = c_i. \tag{31}$$

The solution is unique if and only if the network is minimally connected.

Proof. With the restrictions that $\mu_{ik} = -\mu_{ki}$ for all linked pair ik, (31) constitutes a system of n linear equations with $\frac{1}{2}\sum_{i \in N} d_i$ variables μ_{ik}. Summing up all the n equations, we have

$$0 = \sum_{i<k,\, G_{ik}=1} (\mu_{ik} + \mu_{ki}) = \sum_{i \in N} c_i = 0.$$

Hence, the n linear equations impose at most $(n-1)$ linearly independent conditions.

Viewing (31) in vector form,

$$C\mu = c$$

where C is a $n \times \frac{1}{2}\sum_{i \in N} d_i$ matrix. Note that in each column of C, denoted C_{ij} for $i < j$, there are either no nonzero entries (when $G_{ij} = 0$), or just two nonzero entries: 1 on the i-th row and -1 on the j-th row when $G_{ij} = 1$. Suppose $G_{ij} = 1$. Then, given any subset of individuals S that include i and j, if the rows of C corresponding to S are linearly dependent, these rows must sum to 0: this can be true only if all entries ik with $i \in S$ and $k \notin S$ are zero, implying that S form a component under G, and thus G is not connected if $\#(S) < n$. This is in contradiction with the supposition that G is connected when $\#(S) < n$. Hence, C must have exactly $(n-1)$ linearly independent rows.

Let $\tilde{C}$ and $\tilde{c}$ be the first $(n-1)$ rows of C and c. Then, as $\tilde{C}$ has full row rank, there always exists a solution to $\tilde{C}\mu = \tilde{c}$, and any of the solutions μ must also solve the equation $C\mu = c$. The solution is unique if and only if the component is minimally connected, when there are precisely $(n-1)$ links and thus $\tilde{C}$ is an invertible square matrix.

We can obtain one particular solution using the following algorithm. First, we can arbitrarily select a subset of links that minimally connect the nodes, i.e., the graph restricted to this subset of links is minimally connected. Then, there must exist at least one peripheral node, and we can first easily obtain μ_{ij} for all such peripheral nodes $i \in P_1 := \{k \in N : \ d_k = 1\}$. Then, we can look for new peripheral nodes ignoring the links involving nodes in P_1, and obtain μ_{ij} for all $i \in P_2 := \{k \in N : \ k \notin P_1 \wedge G_{kj} = 1 \text{ for some } j \in P_1\}$ with all previously calculated μ's taken as given. We iterate this process until we exhaust all nodes. Then we are left with a profile of μ that solves (31). ∎

C.3 Proof of Lemma 7

Proof. For general network structures, the analysis is very similar to the above, but there are several complications. As $I_{ij} = \left(e_i, e_j, e_{N_{ij}}\right)$, the transfer rule t_{ij} can be contingent on $e_{N_{ij}} := (e_k)_{k \in N_{ij}}$ in addition to e_i, e_j. Furthermore, as the knowledge of the ex post realization of $e_{N_{ij}}$ brings in extra information about the distribution of non-local endowment realizations, Pareto efficiency requires that t_{ij} be contingent on $e_{N_{ij}}$. Specifically,

$$e_k|_{e_i,e_j,e_{N_{ij}}} \sim \mathcal{N}\left(\frac{\rho}{1+(d_{ij}+1)\rho}\left(e_i + e_j + \sum_{k \in N_{ij}} e_k\right),\ V_{d_{ij}+2}\right) \tag{32}$$

where $d_{ij} := \#(N_{ij})$ and $V_{d_{ij}+2}$ denotes the variance of e_k conditional on observing $(d_{ij}+2)$ endowment realizations.[47]

We again postulate a linear transfer rule: $t_{ij} = \alpha_{ij}e_i - \alpha_{ji}e_j + \sum_{k \in N_{ij}} \beta_{ijk}e_k + \mu_{ij}$, and plug in the postulated form to obtain a system of verification equations. Again, we ignore the verification equations for the state-independent transfers μ, and defer the discussion of

[47]See, for example, Eaton (2007), p116-117.

μ to Section 6.3. After some tedious algebraic transformations, we again arrive at a rather complicated system of linear equations in (α, β) that defines the condition for Pareto efficiency, namely system (33) as shown below.

Lemma 9. *A linear profile of transfer rules $t = (\alpha, \beta, \mu)$ is Pareto efficient if $\forall ij$ s.t. $G_{ij} = 1$,*

$$\begin{cases} \alpha_{ij} & = \frac{1}{2}\left(1 - \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} + \sum_{k \in N_{ij}} \beta_{jki} + \gamma_{ij}\right) \\ \beta_{ijk} & = \frac{1}{2}\Big[\alpha_{ki} - \alpha_{kj} + \sum_{h \in N_{ijk}} \left(\beta_{ihk} - \beta_{jhk}\right) \\ & \quad - \sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ihk} + \sum_{h \in N_{jk} \setminus \overline{N}_i} \beta_{jkh} + \gamma_{ij}\Big] \quad \forall k \in N_{ij} \\ \gamma_{ij} & = \frac{\rho}{1 + (d_{ij}+1)\rho}\Big[\sum_{k \in N_i \setminus \overline{N}_j} \left(\alpha_{ki} - \sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ikh}\right) \\ & \quad - \sum_{k \in N_j \setminus \overline{N}_i} \left(\alpha_{kj} - \sum_{h \in N_{jk} \setminus \overline{N}_i} \beta_{jkh}\right) \\ & \quad - \sum_{k \in N_{ij}} \left(\sum_{h \in N_{ik} \setminus \overline{N}_j} \beta_{ikh} - \sum_{h \in N_{jk} \setminus \overline{N}_i} \beta_{jkh}\right)\Big] \end{cases} \tag{33}$$

Instead of solving for this complicated system directly, we first present an innocuous simplification of it. Due to the possible existence of cycles and superfluous transfers along cycles, this system may in general admit multiple solutions. For example, given a complete triad ijk, we can make a superfluous transfer of a ϵ share of e_i from i to j, j to k and k to i by adding ϵ to α_{ij}, β_{jki}, and subtracting ϵ from α_{ik}. It can then be checked that this operation is indeed superfluous, in the sense that $\left(\alpha_{ij} + \epsilon, \beta_{jki} + \epsilon, \beta_{kji} - \epsilon, \alpha_{ik} - \epsilon\right)$, keeping everything else fixed, still solves the system of equations for Pareto efficiency with the induced final consumption plan left unchanged. Since any amount of superfluous cycles are redundant, we can set $\beta_{ijk} = 0$ for all triads ijk without loss of Pareto efficiency. Hence, in the following, we establish that there exists some vector of strictly bilateral transfer shares $(\alpha^*, \beta^* \equiv \mathbf{0})$ that solves (33) and thus achieves Pareto efficiency. In other words, the strictly bilateral linear transfer rules that we characterize below are the "simplest" Pareto efficient rules in terms of minimizing the sum of state-contingent transfers.

By setting $\beta = \mathbf{0}$, we achieve a significant simplification of (33) and obtain the system (26), which is repeated here for easier reference:

$$\begin{cases} \alpha_{ij} = \frac{1}{2}\left(1 - \sum_{k \in N_i \setminus \{j\}} \alpha_{ik} + \gamma_{ij}\right) & \textbf{(??}.1) \\ 0 = \alpha_{ki} - \alpha_{kj} + \gamma_{ij} \quad \forall k \in N_{ij} & \textbf{(??}.2) \\ \gamma_{ij} = \frac{\rho}{1 + (d_{ij}+1)\rho}\left(\sum_{k \in N_i \setminus \overline{N}_j} \alpha_{ki} - \sum_{k \in N_j \setminus \overline{N}_i} \alpha_{kj}\right) & \textbf{(??}.3) \end{cases} \forall i, j \text{ s.t. } G_{ij} = 1$$

The first equation (26.1) states that the share of e_i transferred from i to j is half of the remaining share after i's transfers to i's other neighbors plus the informational adjustment term between ij. With $\gamma \equiv 0$, which is implied by $\rho = 0$, α will be simply reduced to the local equal sharing rule. The second equation (26.2) requires that the difference in the shares of e_k undertaken by i and j is equal to the informational effect between ij, so that it is indeed optimal for ij to set $\beta_{ijk} = 0$. This confirms again that strict bilaterality ($\beta = \mathbf{0}$) is not an assumption, as (26.2) also incorporates the efficiency requirements for $\beta = \mathbf{0}$. The third equation (26.3) defines the auxiliary variable γ_{ij}. We interpret γ_{ij} as the net informational effect because it is the rate at which locally observed endowment realizations affect the pair ij's joint expectation of non-local endowments. Notice that γ_{ij} is the same across $k \in \overline{N}_{ij}$ because each element of $(e_k)_{k \in \overline{N}_{ij}}$ provides exactly the same amount of information to the linked pair ij for their joint inference on non-local endowments. Given α, $|\gamma_{ij}|$ is decreasing in d_{ij}, indicating that the magnitude of the informational effect (for any single endowment realization) is decreasing in the amount of local information. Below we proceed to show the existence and provide a closed-form characterization of a solution to (26).

We first prove that (26.2) are implied by (26.1) and (26.3). By differencing (26.1) for ki and for kj we get: $\alpha_{ki} - \alpha_{kj} = \gamma_{ki} - \gamma_{kj}$. Hence, in the presence of (26.1) equation (26.2) is equivalent to, for all triads ijk, $\gamma_{ij} + \gamma_{jk} + \gamma_{ki} = 0$. This is reminiscent of the Kirchhoff Voltage Law for electric resistor networks, which states that the sum of voltage differences across any closed cycle must sum to zero. It turns out that the Kirchhoff Voltage Law indeed holds in our setting for any cycle in a general network. ∎

Lemma 10. ***"Kirchhoff Voltage Law"****: $\forall \rho \in \left(-\frac{1}{n-1}, 1\right)$, if (26.1) and (26.3) admit a unique solution (α, γ), this solution also satisfy (26.2); furthermore, given any cycle $i_1 i_2 ... i_m i_1$, γ satisfies the "Kirchhoff Voltage Law" $\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1} = 0$.*

Proof. Intuitively, Pareto optimality requires that ij share equally the net difference in the conditional expectations of *nonlocal* inflow exposures (captured by γ_{ij}) by creating an opposite net difference in their *local* inflow exposures, as specified in equation (26.2). This adjustment guarantees the expectational Borch rule in equation (5), and therefore Pareto efficiency. To see this, notice that conditional expectation and variance of consumption will differ only by a constant across different local states (I_{ij}). Together, this implies that conditional CE's differ only by a constant, as required.

Given the redundancy of (26.2) in the presence of (26.1) and (26.3), we may now conclude that any solution to the system consisting of (26.1) and (26.3) defines a linear and Pareto efficient profile of transfer rules in $\mathcal{T}$. ∎

C.4 Proof of Lemma 8

Proof. Write system (26) in the following form:

$$\begin{cases} 2\alpha_{ij} + \sum_{k\in N_i\setminus\{j\}} \alpha_{ik} - \gamma_{ij} = 1, \; \textcircled{1}_{ij} & \forall G_{ij} = 1 \\ \gamma_{ij} = \frac{\rho}{1+(d_{ij}+1)\rho} \left(\sum_{k\in N_i\setminus \overline{N}_j} \alpha_{ki} - \sum_{k\in N_j\setminus \overline{N}_i} \alpha_{kj}\right), \; \textcircled{3}_{ij} & \forall G_{ij} = 1. \end{cases}$$

This is a system of $2\sum_i d_i$ equations in $2\sum_i d_i$ variables (α, γ). Notice that this system can have at most one solution by Proposition 2, as each distinct solution to the above system will define a distinct consumption plan.

Write system (12) in the following form: $\forall ij$ s.t. $G_{ij} = 1$, and $\forall i \in N$

$$\begin{cases} \alpha_{ji} = \Lambda_j - \frac{\rho}{1-\rho} \left(\sum_{k\in N_i} \alpha_{ki} + \alpha_{ii}\right), \; \textcircled{12}_{ji} & \forall G_{ij} = 1 \\ \alpha_{ii} = \Lambda_i - \frac{\rho}{1-\rho} \left(\sum_{k\in N_i} \alpha_{ki} + \alpha_{ii}\right), \; \textcircled{12}_{ii} & \forall i \in N \\ \alpha_{ii} + \sum_{k\in N_i} \alpha_{ik} = 1, \; \textcircled{13}_{i} & \forall i \in N \end{cases}$$

This is a system of $(\sum_i d_i + 2n)$ equations in $(\sum_i d_i + 2n)$ variables (α, Λ). Suppose that this system has a unique solution. [48]

We now show that there exist $\sum_i d_i$ linearly independent sequences of row operations that produce the tautology "0 = 0". Given that the system $\textcircled{12}\textcircled{13}$ has a unique solution $\left((\alpha_{ij})_{G_{ij}=1}, \alpha\right)$, this will imply that the $(\alpha_{ij})_{G_{ij}=1}$, along with γ defined by $\textcircled{3}$, will also solve system $\textcircled{1}\textcircled{3}$.

Notice that, by the proof of Lemma 10 in Appendix C.6, $\textcircled{1}$ and $\textcircled{3}$ imply that

$$(1+\rho)\,\gamma_{ij} = \rho \left(\sum_{h\in N_i} \alpha_{hi} - \sum_{h\in N_j} \alpha_{hj} + \alpha_{ij} - \alpha_{ji}\right) \quad \textcircled{5}_{ij}.$$

In other words, $\textcircled{5}_{ij}$ can be obtained by a sequence of row operations on $\textcircled{1}$ and $\textcircled{3}$.

Consider a fixed linked pair ij with $i < j$.

By $(1-\rho) \times \left(\textcircled{12}_{ji} - \textcircled{12}_{ij} + \textcircled{12}_{ii} - \textcircled{12}_{jj}\right)$, we have

$$(1-\rho)\left(\alpha_{ji} - \alpha_{ij} + \alpha_{ii} - \alpha_{jj}\right) + 2\rho \left(\sum_{k\in N_i} \alpha_{ki} + \alpha_{ii} - \sum_{k\in N_j} \alpha_{kj} - \alpha_{jj}\right) = 0,$$

[48] It indeed has a unique solution given by Proposition 4.

which is equivalent to

$$(1+\rho)\left(\alpha_{ii}-\alpha_{jj}+\alpha_{ij}-\alpha_{ji}\right)+2\rho\left(\sum_{k\in N_i}\alpha_{ki}-\sum_{k\in N_j}\alpha_{kj}+\alpha_{ij}-\alpha_{ji}\right)=0.$$

Plugging $\textcircled{5}_{ij}$ into the second term above, we have

$$(1+\rho)\left(\alpha_{ii}-\alpha_{jj}\right)+(1-\rho)\left(\alpha_{ji}-\alpha_{ij}\right)+2\left(1+\rho\right)\gamma_{ij}=0,$$

which, divided by $(1+\rho)$ on both sides, is equivalent to

$$\alpha_{ii}-\alpha_{jj}+\alpha_{ji}-\alpha_{ij}+2\gamma_{ij}=0.\textcircled{14}_{ij}$$

By $\textcircled{13}_{i}-\textcircled{1}_{ij}$, we have

$$\alpha_{ii}-\alpha_{ij}+\gamma_{ij}=0.\textcircled{15}_{ij}$$

By $\textcircled{13}_{j}-\textcircled{1}_{ji}$, we have $\alpha_{jj}-\alpha_{ji}+\gamma_{ji}=0$. As $\textcircled{3}_{ij}+\textcircled{3}_{ji}$ implies $\gamma_{ij}+\gamma_{ji}=0$, we have

$$\alpha_{jj}-\alpha_{ji}-\gamma_{ij}=0.\textcircled{16}_{ij}$$

By $\textcircled{14}_{ij}-\textcircled{15}_{ij}+\textcircled{16}_{ij}$, we reach the tautology "0 = 0".

Now, consider $\textcircled{12}_{ji}+\textcircled{12}_{ij}-\textcircled{12}_{ii}-\textcircled{12}_{jj}$, which leads to

$$\alpha_{ji}+\alpha_{ij}-\alpha_{ii}-\alpha_{jj}=0.\textcircled{17}_{ij}$$

Then $\textcircled{17}_{ij}+\textcircled{15}_{ij}+\textcircled{16}_{ij}$ leads to the tautology "0 = 0".

In summary of the above, for each fixed linked pair ij with $i<j$, we have established that

$$\begin{cases}\xi_{ij}: & \frac{1-\rho}{1+\rho}\left(\textcircled{12}_{ji}-\textcircled{12}_{ij}+\textcircled{12}_{ii}-\textcircled{12}_{jj}\right)-\textcircled{13}_{i}+\textcircled{13}_{j}+\zeta_{ij}^{'}\textcircled{1}+\eta_{ij}^{'}\textcircled{3}=\mathbf{0}^{'}.\\ \tilde{\xi}_{ij}: & \textcircled{12}_{ji}+\textcircled{12}_{ij}-\textcircled{12}_{ii}-\textcircled{12}_{jj}+\textcircled{13}_{i}+\textcircled{13}_{j}+\tilde{\zeta}_{ij}^{'}\textcircled{1}+\tilde{\eta}_{ij}^{'}\textcircled{3}=\mathbf{0}^{'}.\end{cases}$$

for some conformable vector $\zeta_{ij},\tilde{\zeta}_{ij},\eta_{ij},\tilde{\eta}_{ij}$. Clearly, the two tautology-generating row operations above are linear independent: any linear combination of the two operations that cancels out $\textcircled{12}_{ji}$ cannot cancel out $\textcircled{12}_{ij}$.

Moreover, $\textcircled{12}_{ij},\textcircled{12}_{ji}$ do not show up in any tautology-generating row operation within

$\left\{\xi_{hk}, \tilde{\xi}_{hk}:\ (i,j) \neq \{h,k\}\right\}$, so $\left\{\xi_{ij}, \tilde{\xi}_{ij}\right\}$ must be linearly independent from $\left\{\xi_{hk}, \tilde{\xi}_{hk}:\ (i,j) \neq \{h,k\}\right\}$.

Hence, we have constructed a set of $\sum_i d_i$ linearly independent tautology-generating row operations $\left\{\xi_{ij}, \tilde{\xi}_{ij}:\ G_{ij}=1,\ i<j\right\}$. ∎

C.5 Proof of Lemma 9

Lemma. 9: *A linear profile of transfer rules $t=(\alpha,\beta,\mu)$ is Pareto efficient if $\forall ij$ s.t. $G_{ij}=1$,*

$$\begin{cases} \alpha_{ij} & = \frac{1}{2}\left(1-\sum_{k\in N_i\backslash\{j\}}\alpha_{ik}+\sum_{k\in N_{ij}}\beta_{jki}+\gamma_{ij}\right) \\ \beta_{ijk} & = \frac{1}{2}\left[\alpha_{ki}-\alpha_{kj}+\sum_{h\in N_{ijk}}\left(\beta_{ihk}-\beta_{jhk}\right)\right. \\ & \quad \left.-\sum_{h\in N_{ik}\backslash\overline{N}_j}\beta_{ihk}+\sum_{h\in N_{jk}\backslash\overline{N}_i}\beta_{jkh}+\gamma_{ij}\right] \quad \forall k\in N_{ij} \\ \gamma_{ij} & = \frac{\rho}{1+(d_{ij}+1)\rho}\left[\sum_{k\in N_i\backslash\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\backslash\overline{N}_j}\beta_{ikh}\right)\right. \\ & \quad -\sum_{k\in N_j\backslash\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\backslash\overline{N}_i}\beta_{jkh}\right) \\ & \quad \left.-\sum_{k\in N_{ij}}\left(\sum_{h\in N_{ik}\backslash\overline{N}_j}\beta_{ikh}-\sum_{h\in N_{jk}\backslash\overline{N}_i}\beta_{jkh}\right)\right] \end{cases} \tag{33}$$

Proof. For each $k\in N_i\backslash\{j\}$, we then have

$$\begin{aligned} \sum_{k\in N_i\backslash\{j\}} t_{ik} &= e_i\sum_{k\in N_i\backslash\{j\}}\alpha_{ik}-\sum_{k\in N_i\backslash\{j\}}\alpha_{ki}e_k+\sum_{k\in N_i\backslash\{j\}}\sum_{h\in N_{ik}}\beta_{ikh}e_h+c_{ij} \\ &= e_i\sum_{k\in N_i\backslash\{j\}}\alpha_{ik}-\sum_{k\in N_{ij}}\alpha_{ki}e_k+\sum_{k\in N_{ij}}\left(\beta_{ikj}e_j+\sum_{h\in N_{ijk}}\beta_{ikh}e_h\right)+\sum_{k\in N_i\backslash\overline{N}_j}\sum_{h\in N_{ijk}}\beta_{ikh}e_h \\ &\quad -\sum_{k\in N_i\backslash\overline{N}_j}\alpha_{ki}e_k+\sum_{k\in N_{ij}}\sum_{h\in N_{ik}\backslash\overline{N}_j}\beta_{ikh}e_h+\sum_{k\in N_i\backslash\overline{N}_j}\sum_{h\in N_{ik}\backslash\overline{N}_j}\beta_{ikh}e_h+c_{ij} \end{aligned}$$

so that

$$\begin{aligned} t_{ij} = &\frac{1}{2}e_i-\frac{1}{2}e_j-\frac{1}{2}\sum_{k\in N_i\backslash\{j\}}\alpha_{ik}e_i+\frac{1}{2}\sum_{k\in N_j\backslash\{i\}}\alpha_{jk}e_j-\frac{1}{2}\sum_{k\in N_{ij}}\left(\beta_{ikj}e_j-\beta_{jki}e_i\right) \\ &+\frac{1}{2}\sum_{k\in N_{ij}}\left[\left(\alpha_{ki}-\alpha_{kj}\right)e_k-\sum_{h\in N_{ijk}}\left(\beta_{ikh}-\beta_{jkh}\right)e_h\right] \\ &-\frac{1}{2}\sum_{k\in N_i\backslash\overline{N}_j}\sum_{h\in N_{ijk}}\beta_{ikh}e_h+\frac{1}{2}\sum_{k\in N_j\backslash\overline{N}_i}\sum_{h\in N_{ijk}}\beta_{jkh}e_h \\ &-\frac{1}{2}\frac{\rho}{1+(d_{ij}+1)\rho}\left(e_i+e_j+\sum_{k\in N_{ij}}e_k\right)\sum_{k\in N_{ij}}\left(\sum_{h\in N_{ik}\backslash\overline{N}_j}\beta_{ikh}-\sum_{h\in N_{jk}\backslash\overline{N}_i}\beta_{jkh}\right) \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\frac{\rho}{1+(d_{ij}+1)\rho}\left(e_i+e_j+\sum_{k\in N_{ij}}e_k\right)\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)\\
&-\frac{1}{2}\frac{\rho}{1+(d_{ij}+1)\rho}\left(e_i+e_j+\sum_{k\in N_{ij}}e_k\right)\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\\
=&\,\frac{1}{2}\left\{1-\sum_{k\in N_i\setminus\{j\}}\alpha_{ik}+\sum_{k\in N_{ij}}\beta_{jki}+\frac{\rho}{1+(d_{ij}+1)\rho}\left[\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)\right.\right.\\
&\left.\left.-\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)-\sum_{k\in N_{ij}}\left(\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right]\right\}\cdot e_i\\
&-\frac{1}{2}\left\{1-\sum_{k\in N_j\setminus\{i\}}\alpha_{jk}+\sum_{k\in N_{ij}}\beta_{ikj}+\frac{\rho}{1+(d_{ij}+1)\rho}\left[\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right.\right.\\
&\left.\left.-\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)-\sum_{k\in N_{ij}}\left(\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)\right]\right\}\cdot e_j\\
&+\frac{1}{2}\sum_{k\in N_{ij}}\left\{\alpha_{ki}-\alpha_{kj}+\sum_{h\in N_{ijk}}\left(\beta_{ihk}-\beta_{jhk}\right)-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ihk}+\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right.\\
&+\frac{\rho}{1+(d_{ij}+1)\rho}\left[\sum_{k\in N_i\setminus\overline{N}_j}\left(\alpha_{ki}-\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}\right)-\sum_{k\in N_j\setminus\overline{N}_i}\left(\alpha_{kj}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right.\\
&\left.\left.-\sum_{k\in N_{ij}}\left(\sum_{h\in N_{ik}\setminus\overline{N}_j}\beta_{ikh}-\sum_{h\in N_{jk}\setminus\overline{N}_i}\beta_{jkh}\right)\right]\right\}\cdot e_k+C_{ij}
\end{aligned}
$$

The last equality is obtained by collecting terms and switching summand indexes. ∎

C.6 Proof of Lemma 10

Lemma. 10: ***"Kirchhoff Voltage Law"**: $\forall\rho\in\left(-\frac{1}{n-1},1\right)$, if (26.1) and (26.3) admit a unique solution (α,γ), this solution also satisfy (26.2); furthermore, given any cycle $i_1 i_2 ... i_m i_1$, γ satisfies the "Kirchhoff Voltage Law" $\gamma_{i_1 i_2}+\gamma_{i_2 i_3}+...+\gamma_{i_m i_1}=0$.*

Proof. We begin by proving the first part, which only involves triads. We rewrite (26) in the following way:

$$\begin{cases} 2\alpha_{ij} + \sum_{k\in N_i\setminus\{j\}} \alpha_{ik} - \gamma_{ij} = 1, \quad \forall G_{ij} = 1 & \textcircled{1} \\ \alpha_{ki} - \alpha_{kj} + \gamma_{ij} = 0 \quad \forall k \in N_{ij}, \quad \forall G_{ij} = 1; & \textcircled{2} \\ \gamma_{ij} = \frac{\rho}{1+(d_{ij}+1)\rho}\left(\sum_{k\in N_i\setminus\overline{N}_j} \alpha_{ki} - \sum_{k\in N_j\setminus\overline{N}_i} \alpha_{kj}\right), \quad \forall G_{ij} = 1; & \textcircled{3} \end{cases}$$

In matrix form we write

$$\begin{bmatrix} A \\ M \end{bmatrix} \begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} b \\ \mathbf{0} \end{pmatrix} \qquad \begin{matrix} \textcircled{1} \wedge \textcircled{3} \\ \textcircled{2} \end{matrix}$$

where α, γ are both $\sum_i d_i$-dimensional vectors, A is a $(2\sum_i d_i) \times (2\sum_i d_i)$ square matrix, $b := \begin{pmatrix} \mathbf{1}_{\sum_i d_i} \\ \mathbf{0}_{\sum_i d_i} \end{pmatrix}$ is a $(2\sum_i d_i)$-dimensional vector, M is a $\left(\sum_{G_{ij}=1} d_{ij}\right) \times (2\sum_i d_i)$ rectangular matrix, and $\mathbf{0}$ is a $\left(\sum_{G_{ij}=1} d_{ij}\right)$-dimensional vector. The upper block $A\begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = b$ corresponds to equations in ① and ③, while the lower block $M\begin{pmatrix} \alpha \\ \gamma \end{pmatrix} = \mathbf{0}$ corresponds to equations in ②.

Note that $③_{ij}$ in the definition above is not written in its canonical form, i.e., it is not written in such way that the LHS of the equality sign is a linear combination of unknown variables (α, γ) while the right-hand side (RHS) is a constant scalar. In the following, we interpret any written linear equation to be representative of the underlying canonical form obtained by moving all linear combinations of (α, γ) on the RHS of "=" to the LHS (left-hand side) while moving all constants on the LHS to the RHS. For example, we interpret $③_{ij}$ to represent a canonical form such that the coefficient before the unknown variable γ_{ij} is 1 and the coefficient before α_{ki} for some $k \in N_i\setminus\overline{N}_j$ to be $-\frac{\rho}{1+(d_{ij}+1)\rho}$.[49]

Given that the system consisting of ① and ③ admit a unique solution, its coefficient matrix A and its augmented matrix $\tilde{A} = [A|\, b]$ must have full rank $2\sum_i d_i$. To prove that the unique solution of ① and ③ also satisfies ②, it suffices to show that the augmented matrix for the system of ①, ③ and ②

$$\left[\begin{array}{c|c} A & b \\ M & \mathbf{0} \end{array}\right]$$

still have rank $2\sum_i d_i$. We show this by demonstrating the existence of $\sum_{G_{ij}=1} d_{ij}$ nonzero and

[49]This convention should resolve any ambiguity about the signs of coefficients before (α, γ) in all the equations written out thereafter.

linearly independent vector $\xi \in \mathbb{R}^{2\sum_i d_i+\sum_{G_{ij}=1} d_{ij}}$ such that

$$\xi' \begin{bmatrix} A & b \\ M & \mathbf{0} \end{bmatrix} = (0, 0, ..., 0)_{2\sum_i d_i+1} .$$

We first fix any linked triad ijk.

Multiplying $\textcircled{3}_{ij}$ (the ij-th equation in $\textcircled{3}$) with $(1 + (d_{ij} + 1)\rho)$, we obtain $[1 + (d_{ij} + 1)\rho]\gamma_{ij} = \rho\left(\sum_{h\in N_i\setminus\overline{N}_j} \alpha_{hi} - \sum_{h\in N_j\setminus\overline{N}_i} \alpha_{hj}\right)$, which is equivalent to

$$[1 + (d_{ij} + 1)\rho]\gamma_{ij} = \rho\left[\sum_{h\in N_i} \alpha_{hi} - \sum_{h\in N_j} \alpha_{hj} - \sum_{h\in N_{ij}} (\alpha_{hi} - \alpha_{hj}) - \alpha_{ji} + \alpha_{ij}\right] \quad \textcircled{4}_{ij}.$$

Adding $\textcircled{2}_{ijh}$ for all $h \in N_{ij}\setminus\{k\}$ to $\textcircled{4}_{ij}$, we get

$$[1 + (d_{ij} + 1)\rho]\gamma_{ij} = \rho\left[\sum_{h\in N_i} \alpha_{hi} - \sum_{h\in N_j} \alpha_{hj} + (d_{ij} - 1)\gamma_{ij} - \alpha_{ki} + \alpha_{kj} - \alpha_{ji} + \alpha_{ij}\right]$$

which is equivalent to

$$(1 + 2\rho)\gamma_{ij} = \rho\left(\sum_{h\in N_i} \alpha_{hi} - \sum_{h\in N_j} \alpha_{hj} + \alpha_{kj} - \alpha_{ki} + \alpha_{ij} - \alpha_{ji}\right) \quad \textcircled{5}_{ij}.$$

Summing up $\textcircled{5}_{ij}, \textcircled{5}_{jk}, \textcircled{5}_{ki}$, we have

$$\begin{aligned}(1 + 2\rho)\left(\gamma_{ij} + \gamma_{jk} + \gamma_{ki}\right) &= \rho\left[(\alpha_{kj} - \alpha_{ki} + \alpha_{ij} - \alpha_{ji}) + (\alpha_{ik} - \alpha_{ij} + \alpha_{jk} - \alpha_{kj})\right.\\ &\qquad \left. + (\alpha_{ji} - \alpha_{jk} + \alpha_{ki} - \alpha_{ik})\right]\\ &= 0\end{aligned}$$

For $n = 3$ and $\rho > -\frac{1}{2}$, or for $n \geq 4$, we have $1 + 2\rho > 0$ and thus

$$\gamma_{ij} + \gamma_{jk} + \gamma_{ki} = 0. \quad \textcircled{6}_{ijk}$$

Alternatively, taking $\textcircled{1}_{ki} - \textcircled{1}_{kj} + \textcircled{2}_{ijk}$, we obtain $\gamma_{ij} - \gamma_{kj} + \gamma_{ki} = 0$. By $\textcircled{3}_{jk} + \textcircled{3}_{kj}$, we have $\gamma_{jk} + \gamma_{kj} = 0$ and thus

$$\gamma_{ij} + \gamma_{jk} + \gamma_{ki} = 0. \quad \textcircled{7}_{ijk}$$

Then $\textcircled{6}_{ijk} - \textcircled{7}_{ijk}$ leads to the tautology "0 = 0". Let $\xi^{ijk} \in \mathbb{R}^{2\sum_i d_i + \sum_{G_{ij}=1} d_{ij}}$ be a vector that characterizes all the row operations conducted above. Clearly

$$\left(\xi^{ijk}\right)' \begin{bmatrix} A & b \\ M & \mathbf{0} \end{bmatrix} = \mathbf{0}'.$$

Notice that we can obtain one ξ^{ijk} for each ordered triad (i, j, k). Clearly each ξ^{ijk} is nonzero: in particular, the entries of ξ that correspond to equations $\textcircled{1}_{ki}$ and $\textcircled{1}_{kj}$ must be nonzero, $\xi_{\textcircled{1}_{ki}} \neq 0$, $\xi_{\textcircled{1}_{kj}} \neq 0$, because $\textcircled{1}_{ki}, \textcircled{1}_{kj}$ are used to obtain $\textcircled{7}_{ijk}$ and nowhere else.

Fixing k, for a row operation in question $\xi^{i_1 i_2 i_3}$, coefficients corresponding to $\textcircled{1}_{kh}$ for $h \in N_k$ may be nonzero only if $i_3 = k$. Hence, $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$ must be linearly independent from $\left\{\xi^{i_1 i_2 i_3} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1, i_3 \neq k\right\}$. We now consider $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$. Notice that $\textcircled{1}_{ki}, \textcircled{1}_{kj}$ show up in the form of "$\textcircled{1}_{ki} - \textcircled{1}_{kj}$" during the process. Hence, summing up along general cycles[50] is the only possible type of row operations that can cancel out all coefficients before $\textcircled{1}_{ki}$ for all $i \in N_k$. However, this operation does not lead to the tautology $\left(\mathbf{0}', 0\right)$, because the coefficients before $\textcircled{2}_{i_1 i_2 k}, ..., \textcircled{2}_{i_m i_1 k}$ are all kept nonzero. (Notice that these only show up in $\xi^{i_1 i_2 k}$ in the step leading to $\textcircled{7}_{ijk}$ and nowhere else). Hence, no nontrivial linear combination of $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$ is zero, so $\left\{\xi^{i_1 i_2 k} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$ is linearly independent. In summary, we conclude that $\left\{\xi^{i_1 i_2 i_3} : i_1, i_2 \in N_k, G_{i_1 i_2} = 1\right\}$ are linearly independent, so we have established the existence of $\sum_{G_{ij}=1} d_{ij}$ nonzero and linearly independent vector $\xi \in \mathbb{R}^{2\sum_i d_i + \sum_{G_{ij}=1} d_{ij}}$.

We now prove the second part, the statement for cycles of any size. Note that we still have $\textcircled{5}_{ij}$: $(1+\rho)\gamma_{ij} = \rho\left(\sum_{h \in N_i} \alpha_{hi} - \sum_{h \in N_j} \alpha_{hj} + \alpha_{ij} - \alpha_{ji}\right)$. Given any cycle $i_1 i_2 ... i_m i_1$, summing up $\textcircled{5}_{i_1 i_2}, \textcircled{5}_{i_2 i_3}, ..., \textcircled{5}_{i_m i_1}$, we have

$$(1+\rho)\left(\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1}\right) = \rho\left(\alpha_{i_1 i_2} + ... + \alpha_{i_m i_1} - \alpha_{i_2 i_1} - ... - \alpha_{i_1 i_m}\right) \quad \textcircled{10}$$

By $\textcircled{1}_{i_1 i_2} - \textcircled{1}_{i_2 i_1}$ and $\gamma_{ij} + \gamma_{ji} = 0$, we have $\alpha_{i_1 i_2} - \alpha_{i_2 i_1} = \sum_{h \in N_{i_2}} \alpha_{i_2 h} - \sum_{h \in N_{i_1}} \alpha_{i_1 h} + 2\gamma_{i_1 i_2}$. Summing over $i_1 i_2, ..., i_m i_1$,

$$\alpha_{i_1 i_2} + ... + \alpha_{i_m i_1} - \alpha_{i_2 i_1} - ... - \alpha_{i_1 i_m} = 2\left(\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1}\right) \quad \textcircled{11}$$

Then $\textcircled{10} + \rho \times \textcircled{11}$ gives $(1-\rho)\left(\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1}\right) = 0$. For $\rho < 1$, we have $\gamma_{i_1 i_2} + \gamma_{i_2 i_3} + ... + \gamma_{i_m i_1} = 0$. ∎

[50] By general cycles we mean cycles that may involve "self cycles" of the form "$i_1 i_2 i_1$".

C.7 Uniqueness in Minimally Connected Networks

Proposition 13. *Under the independent CARA-Normal setting, if the network is minimally connected, then there is a unique profile of transfer rules in $\mathcal{T}^*$ that is Pareto efficient, and it takes the form of the local equal sharing rule.*

Proof. Consider minimally connected network G. For Pareto efficiency, we need for all linked pair ij

$$\frac{\mathbb{E}_{ij}\left[u_i'\left(x_i\right)\right]}{\mathbb{E}_{ij}\left[u_j'\left(x_j\right)\right]} = c_{ij}.$$

As the network is minimally connected, we have $N_{ij} = \emptyset$. Notice that

$$\mathbb{E}\left[re^{-r\left(e_i - t_{ij} - \sum_{k\in N_i\setminus\{j\}} t_{ik}(e_i,e_k)\right)}\middle| e_i, e_j\right] = c_{ij}\mathbb{E}\left[re^{-r\left(e_j + t_{ij} - \sum_{h\in N_j\setminus\{i\}} t_{jh}(e_j,e_h)\right)}\middle| e_i, e_j\right].$$

[51]By independence,

$$\begin{aligned}
&\mathbb{E}\left[re^{-r\left(e_i - t_{ij} - \sum_{k\in N_i\setminus\{j\}} t_{ik}(e_i,e_k)\right)}\middle| e_i\right] = c_{ij}\mathbb{E}\left[re^{-r\left(e_j + t_{ij} - \sum_{h\in N_j\setminus\{i\}} t_{jh}(e_j,e_h)\right)}\middle| e_j\right] \\
\Leftrightarrow & e^{-r(e_i - t_{ij})}\cdot \prod_{k\in N_i\setminus\{j\}} \mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right] = c_{ij}e^{-r(e_j+t_{ij})}\cdot \prod_{h\in N_j\setminus\{i\}} \mathbb{E}\left[e^{rt_{jh}(e_j,e_h)}\middle| e_j\right] \\
\Leftrightarrow & e_i - t_{ij} - \frac{1}{r}\sum_{k\in N_i\setminus\{j\}} \ln \mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right] = e_j + t_{ij} - \frac{1}{r}\sum_{h\in N_j\setminus\{i\}} \ln \mathbb{E}\left[e^{rt_{jh}(e_j,e_h)}\middle| e_j\right] - \frac{1}{r}\ln c_{ij} \\
\Leftrightarrow & t_{ij} = \frac{1}{2}e_i - \frac{1}{2}e_j - \frac{1}{2r}\sum_{k\in N_i\setminus\{j\}} \ln \mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right] + \frac{1}{2r}\sum_{h\in N_j\setminus\{i\}} \ln \mathbb{E}\left[e^{rt_{jh}(e_j,e_h)}\middle| e_j\right] + \frac{1}{2r}\ln c_{ij} \qquad (34) \\
& = \frac{1}{2}e_i - \frac{1}{2}e_j - \frac{1}{2r}\sum_{k\in N_i\setminus\{j\}} \ln T_{ik} + \frac{1}{2r}\sum_{h\in N_j\setminus\{i\}} \ln T_{jh} + \frac{1}{2r}\ln c_{ij}
\end{aligned}$$

where

$$T_{ik} := \mathbb{E}\left[e^{rt_{ik}(e_i,e_k)}\middle| e_i\right].$$

Then, taking conditional expectations of (34), we have

$$\begin{aligned}
T_{ij} &= e^{r\left(\frac{1}{2}e_i + \frac{1}{2}r\sigma^2 - \frac{1}{2r}\sum_{k\in N_i\setminus\{j\}} \ln T_{ik} + \frac{1}{2r}\ln \alpha_{ij}\right)}\cdot \mathbb{E}\left[e^{r\frac{1}{2r}\sum_{h\in N_j\setminus\{i\}} \ln T_{jh}}\middle| e_i\right] \\
&= e^{r\left(\frac{1}{2}e_i + \frac{1}{2}r\sigma^2 - \frac{1}{2r}\sum_{k\in N_i\setminus\{j\}} \ln T_{ik} + \frac{1}{2r}\ln \alpha_{ij}\right)}\cdot \prod_{h\in N_j\setminus\{i\}} \mathbb{E}\left[T_{jh}^{\frac{1}{2}}\right]
\end{aligned}$$

[51]We hope the unfortunate notational coincidence of the endowment vector e and the natural exponential power $e^{\cdot}$ will not result in any confusion.

and

$$\frac{1}{r}\ln T_{ij} = \frac{1}{2}e_i + \frac{1}{2}r\sigma^2 - \frac{1}{2}\sum_{k\in N_i\setminus\{j\}}\frac{1}{r}\ln T_{ik} + \frac{1}{2r}\ln\alpha_{ij} + \sum_{h\in N_j\setminus\{i\}}\ln\mathbb{E}\left[T_{jh}^{\frac{1}{2}}\right].$$

Writing $\tilde{T}_{ij} := \frac{1}{r}\ln T_{ij}$, we have

$$\tilde{T}_{ij} = \frac{1}{2}e_i - \frac{1}{2}\sum_{k\in N_i\setminus\{j\}}\tilde{T}_{ik} + c_{ij}$$

$\Rightarrow$

$$\sum_{j\in N_i}\tilde{T}_{ij} = \frac{d_i}{2}e_i - \frac{1}{2}\cdot(d_i-1)\sum_{j\in N_i}\tilde{T}_{ik} + \sum_{j\in N_i}c_{ij}$$

$\Rightarrow$

$$\sum_{j\in N_i}\tilde{T}_{ij} = \frac{d_i}{d_i+1}e_i + \frac{2}{d_i+1}\sum_{j\in N_i}c_{ij}$$

$\Rightarrow$

$$\tilde{T}_{ij} = \frac{1}{2}e_i - \frac{1}{2}\sum_{k\in N_i\setminus\{j\}}\tilde{T}_{ik} + c_{ij} = \frac{1}{2}e_i - \frac{1}{2}\sum_{k\in N_i}\tilde{T}_{ik} + \frac{1}{2}\tilde{T}_{ij} + c_{ij}$$

$\Rightarrow$

$$\frac{1}{2}\tilde{T}_{ij} = \frac{1}{2}\left(e_i - \frac{d_i}{d_i+1}e_i - \frac{2}{d_i+1}\sum_{k\in N_i}c_{ik}\right) + c_{ij}$$

$\Rightarrow$

$$\tilde{T}_{ij} = \frac{1}{d_i+1}e_i - \frac{1}{d_i+1}\sum_{k\in N_i}c_{ik} + c_{ij}$$

Hence, by (34), we have

$$\begin{aligned}
t_{ij} &= \frac{1}{2}e_i - \frac{1}{2}e_j - \frac{1}{2}\sum_{k\in N_i\setminus\{j\}}\left(\frac{1}{d_i+1}e_i - \frac{1}{d_i+1}\sum_{k'\in N_i}c_{ik'} + c_{ik}\right) \\
&\quad + \frac{1}{2}\sum_{h\in N_j\setminus\{i\}}\left(\frac{1}{d_j+1}e_j - \frac{1}{d_j+1}\sum_{h'\in N_j}c_{jh'} + c_{jh'}\right) + \frac{1}{2r}\ln\alpha_{ij} \\
&= \frac{1}{2}\left(1 - \frac{d_i-1}{d_i+1}\right)e_i - \frac{1}{2}\left(1 - \frac{d_i-1}{d_i+1}\right)e_j + C_{ij} \\
&= \frac{1}{d_i+1}e_i - \frac{1}{d_i+1}e_j + C_{ij}.
\end{aligned}$$

∎

C.8 Linear Pareto Efficient Transfer Shares for Boundary Correlation Parameters

Proposition 14.

- *for* $\rho = -\frac{1}{n-1}$ *and any* G *such that* $\max_{i \in N} d_i < n-1$, *a Pareto efficient profile of transfer rules is given by Proposition 4*
- *For* $\rho = -\frac{1}{n-1}$ *and any network structure* G *such that* $\max_{i \in N} d_i = n-1$, *let* i^* *be any individual with* $d_{i^*} = n-1$. *Then a Pareto efficient profile of transfer rules is given by*

$$\alpha_{ji^*} = 1, \quad \alpha_{i^*j} = \alpha_{jk} = 0, \quad \forall j, k \in N \backslash \{i^*\}.$$

- *For* $\rho = 1$ *and any network structure* G, *any profile of transfer rules that satisfies the Kirchhoff Circuit Law as defined below is Pareto efficient:*

$$\sum_{j \in \overline{N}_i} \alpha_{ij} = \sum_{j \in \overline{N}_i} \alpha_{ji} \quad \forall i \in N.$$

Proof. For $\rho = -\frac{1}{n-1}$ and G s.t. $\max_{i \in N} d_i = n-1$, the profile of transfer rules given above attains zero variance in consumption for each individual, and is thus Pareto efficient. For $\rho = 1$, any profile of transfer rules that satisfies the Kirchhoff Circuit Law achieves the same profile of consumption plan as the null transfer (autarky), which is clearly Pareto efficient. ∎

C.9 Welfare Comparative Statics w.r.t. ρ

Let $TVar_\rho(\alpha)$ denote the total variance under correlation ρ and any generic transfer shares α, and let $\alpha^*(\rho)$ denote the Pareto efficient transfer shares under correlation $\rho \in (-\frac{1}{n-1}, 1)$.

We first show that $TVar_\rho(\alpha^*(\rho))$ is increasing in ρ on a neighborhood around $\rho = 0$. Specifically, by the Envelope Theorem,

$$\begin{aligned}\frac{d}{d\rho}\left[TVar_\rho(\alpha^*(\rho))\right] = \frac{d}{d\rho}TVar_\rho(\alpha)\bigg|_{\alpha^*(\rho)} &= \frac{d}{d\rho}\sum_i \sum_{j \in N_i} \alpha_{ii}\alpha_{ji}Cov_\rho(e_i, e_j)\bigg|_{\alpha^*(\rho)} \\ &= \sum_i \sum_{j \in N_i} \alpha^*_{ii}(\rho)\alpha^*_{ji}(\rho) > 0\end{aligned}$$

if $\alpha^*_{ii}(\rho) > 0$ and $\alpha^*_{ji}(\rho) > 0$ for all i. When $\rho = 0$, $\alpha^*(\rho)$ is given by the local equal sharing rule, which implies that $\alpha_{ii} = \alpha_{ij} = \frac{1}{d_i+1} > 0$ for all i. Hence, $\frac{d}{d\rho}\left[TVar_\rho(\alpha^*(\rho))\right] > 0$ at $\rho = 0$, so $TVar_\rho(\alpha^*(\rho))$ must be increasing in ρ on a neighborhood around 0.

Next, we show in the star network that the total variance is an increasing and concave function of ρ.[52] Based on the transfer shares obtained in Section 4.1, we can derive:

$$TVar_\rho(\alpha^*(\rho)) = \frac{2(1-\rho)^2 + n^3\rho - n(1-5\rho)(1-\rho) + n^2(1-2\rho+3\rho^2)}{n(2+n\rho)}$$

and

$$\frac{d}{d\rho}TVar_\rho(\alpha^*(\rho)) = \frac{(n-1)(8(1-\rho) - 2n(1-6\rho+\rho^2) + n^2(1+3\rho^2))}{n(2+n\rho)^2} > 0$$

and furthermore

$$\frac{d^2}{d\rho^2}TVar_\rho(\alpha^*(\rho)) = -\frac{2(n-2)^2(n-1)(n+2)}{n(2+n\rho)^3} < 0.$$

Hence, $TVar_\rho(\alpha^*(\rho))$ must be increasing and concave in ρ for any values of n and $\rho \in [-1/(n-1), 1]$.

In Figure 4, we plot out the total variance against the correlation parameter ρ for $n = 10$. Plots for other values of n are very similar.

C.10 Proof of Proposition 9

Proof. Condition (a) implies that the contractibility constraints encoded in Q are equivalent to the "local information constraints", characterized by the common neighborhoods as in Section (3), under the informational network G'. Hence, no feasible consumption plan subject to the contractibility constraints Q can strictly Pareto dominate $x^*\left(G'\right)$.

It remains to show that $x^*\left(G'\right)$ is feasible under contractibility constraints Q on network G. Specifically, consider each link $ij \in G' \backslash G$. As ij is not directly linked in G, the net transfer $t^*_{ij}\left(G'\right) = \alpha^*_{ij}\left(G'\right) e_i - \alpha^*_{ji}\left(G'\right) e_j + \mu^*_{ij}$ cannot be directly transferred between individual i and individual j. However, by Condition (b), there exists a path of individuals in G, in the form of $i = k_0 - k_1 - ... - k_m = j$, such that $\{i, j\} \subseteq \overline{N}_{k_h}\left(G'\right)$ for all $h = 0, ..., m$. Hence, $\{i, j\} \subseteq \overline{N}_{k_h k_{h+1}}\left(G'\right)$ for all $h = 0, ..., m-1$. Now, define

$$\hat{t}_{k_h k_{h+1}}\left(G'\right) := \alpha^*_{ij}\left(G'\right) e_i - \alpha^*_{ji}\left(G'\right) e_j + \mu^*_{ij} \quad \text{for all } h = 0, ..., m-1,$$

[52] Notice that, for star network, the center individual's exposure share of own endowment shock can be negative for certain values of n and ρ.

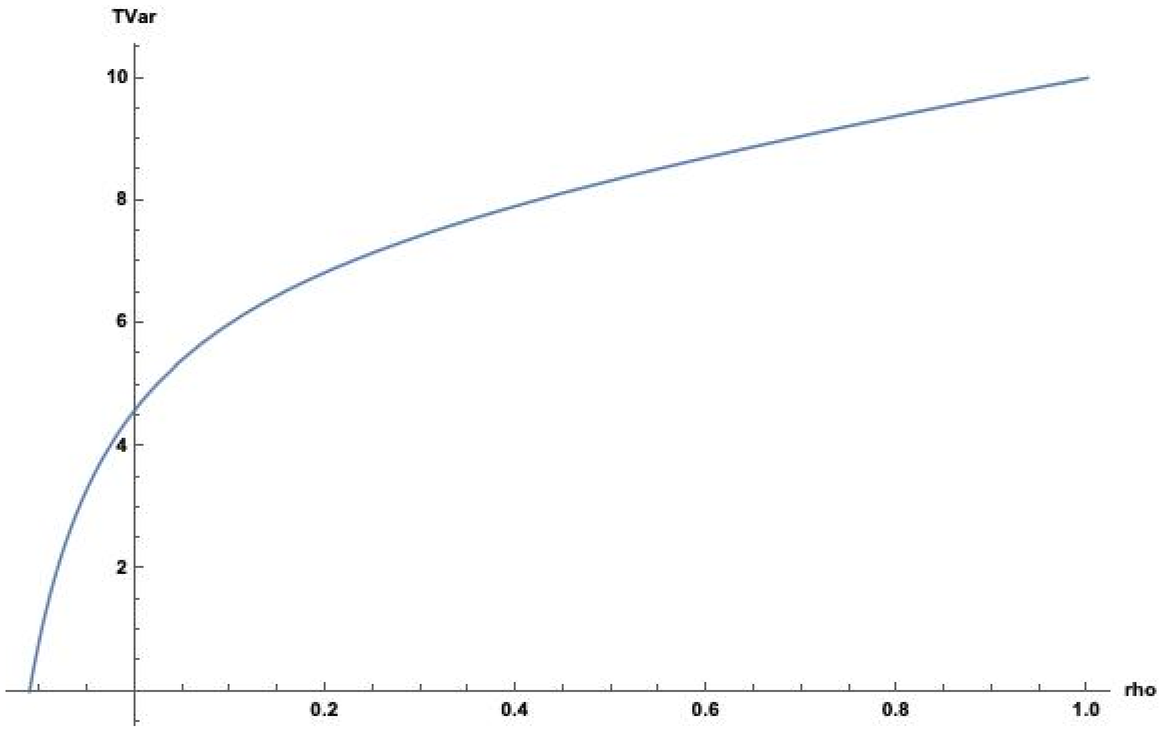


Figure 4: Total Variance vs Correlation Parameter

and

$$\bar{t}_{k_h k_{h+1}} := t^*_{k_h k_{h+1}} \left(G^{'}\right) + \hat{t}_{k_h k_{h+1}} \left(G^{'}\right)$$
$$\bar{t}_{ij} := 0$$

Clearly, the transfer profile $\bar{t}$ still satisfy the contractibility constraints Q, as $\{i, j\} \subseteq \overline{N}_{k_h k_{h+1}} \left(G^{'}\right) = Q_{ij} (G)$ by Condition (a). Now, it is easy to see that the new transfer profile $\bar{t}$ induces the same consumption plan $x^* \left(G^{'}\right)$. However, there is no more direct transfer between individuals i and j, who are not physically linked in G. In the meantime, as $G_{k_h k_{h+1}} = 1$ for all $h = 0, ..., m-1$, we have not added any transfer between pairs of individuals who are *not* originally linked in G.

By induction on all such pairs of individuals $G^{'} \backslash G$, which must terminate in finite steps, we conclude that there exists a transfer profile t^{**} that: (i) satisfies the contractibility constraints Q; (ii) induces the same consumption plan $x^* \left(G^{'}\right)$; and (iii) respects the physical transfer network G, i.e., $t^{**}_{ij} \neq \mathbf{0}$ only if $G_{ij} = 1$. ∎

C.11 Proof of Proposition 10

Proof. For each ordered pair ij such that $\overleftarrow{G}_{ij} = 1$, define the following adjusted local equal sharing rule,

$$t_{ij}^{*}\left(\overleftarrow{G}\right) := \frac{1}{d_j\left(\overleftarrow{G}\right)+1} e_j,$$

as the transfer of j's endowment shock to individual i. Then $t^{*}\left(\overleftarrow{G}\right)$ would lead to the consumption plan $x_i^{*}\left(\overleftarrow{G}\right)$ as defined in (21):

$$x_i = e_i + \sum_{j:\overleftarrow{G}_{ij}=1} t_{ij}^{*}\left(\overleftarrow{G}\right) - \sum_{j:\overleftarrow{G}_{ji}=1} t_{ji}^{*}\left(\overleftarrow{G}\right) = x_i^{*}\left(\overleftarrow{G}\right).$$

For each $ij \in G\backslash\overleftarrow{G}$, there exists a path of individuals $i = k_0k_1...k_m = j$ in G, such that $j \in Q_{k_hk_{h+1}}$ for all $h = 0, ..., m-1$. Define

$$\bar{t}_{k_hk_{h+1}} := t_{k_hk_{h+1}}^{*}\left(\overleftarrow{G}\right) + \frac{1}{d_j\left(\overleftarrow{G}\right)+1} e_j \text{ for } h = 0, ..., m-1,$$

$$\bar{t}_{k_hk_{h+1}} := 0.$$

It is straightforward to see that $\bar{t}$ induces the same consumption plan $x^{*}\left(\overleftarrow{G}\right)$, satisfies the contractibility constraints Q and no longer involves direct transfer between ij. By induction on the set $G\backslash\overleftarrow{G}$, we conclude that there exists a physically feasible linear transfer profile $t^{**}(G)$ that satisfies the contractibility constraints Q and induces the consumption plan $x^{*}\left(\overleftarrow{G}\right)$.

We now show that $x_i^{*}\left(\overleftarrow{G}\right)$ achieves constrained Pareto efficiency subject to the contractibility constraints Q. Fix any $ij \in G$. By the definition of $\overleftarrow{G}$, it is easy to prove that $k \in N_i\left(\overleftarrow{G}\right) \cap Q_{ij}$ implies that $k \in N_j\left(\overleftarrow{G}\right)$, and thus

$$N_i\left(\overleftarrow{G}\right) \cap Q_{ij} = N_{ij}\left(\overleftarrow{G}\right) \cap Q_{ij} = N_j\left(\overleftarrow{G}\right) \cap Q_{ij}.$$

Hence, the difference in the local certainty equivalents for individuals i and j, conditional on subvector of endowment realizations that t_{ij} can be contingent on, namely $e_{\overline{Q}_{ij}} = \left(e_i, e_j, e_{Q_{ij}}\right)$, is given by

$$CE\left(x_i^{*}\left(\overleftarrow{G}\right)\middle| e_{\overline{Q}_{ij}}\right) - CE\left(x_j^{*}\left(\overleftarrow{G}\right)\middle| e_{\overline{Q}_{ij}}\right)$$

$$
\begin{aligned}
=&\frac{1}{d_i\left(\overleftarrow{G}\right)+1}e_i+\frac{1}{d_j\left(\overleftarrow{G}\right)+1}e_j+\sum_{k\in N_i\left(\overleftarrow{G}\right)\cap Q_{ij}}\frac{1}{d_k\left(\overleftarrow{G}\right)+1}e_k\\
&+\sum_{k\in N_i\left(\overleftarrow{G}\right)\setminus Q_{ij}}\frac{1}{d_k\left(\overleftarrow{G}\right)+1}\mathbb{E}\left[e_k|\, e_{\overline{Q}_{ij}}\right]+C_i\\
&-\frac{1}{d_i\left(\overleftarrow{G}\right)+1}e_i-\frac{1}{d_j\left(\overleftarrow{G}\right)+1}e_j-\sum_{k\in N_j\left(\overleftarrow{G}\right)\cap Q_{ij}}\frac{1}{d_k\left(\overleftarrow{G}\right)+1}e_k\\
&-\sum_{k\in N_j\left(\overleftarrow{G}\right)\setminus Q_{ij}}\frac{1}{d_k\left(\overleftarrow{G}\right)+1}\mathbb{E}\left[e_k|\, e_{\overline{Q}_{ij}}\right]-C_j\\
=&C_i-C_j,
\end{aligned}
\tag{35}
$$

as $\mathbb{E}\left[e_k|\, e_{\overline{Q}_{ij}}\right]=0$ for any $k\notin\overline{Q}_{ij}$ due to the independence of endowments, which is essential for this result. As equation (35), an adapted version of (6), represents the FOC's for constrained Pareto efficiency subject to the contractibility constraints Q, we conclude that $x_i^*\left(\overleftarrow{G}\right)$, as well as $t^{**}(G)$, are constrained Pareto optimal. ∎

C.12 Proof of Proposition 11

Proof.

(a) Global Communication

Under the global communication protocol, the *first-best* consumption plan induced by global equal sharing can be achieved.

Specifically, let each individual i submits ex post a public report m_i of her own endowment realization e_i. As the whole vector of reports m is globally observable, for any linked pair ij, they make make their transfer contract t_{ij} effectively contingent on $I_{ij}=(e_k)_{k\in\overline{N}_{ij}}$ and m.

Consider the following specification of t_{ij}:

$$t_{ij}(I_{ij},m)=\tilde{t}_{ij}\left(I_{ij},m_{N\setminus\overline{N}_{ij}}\right)+|e_i-m_i|-|e_j-m_j|,$$

where $m_{N\setminus\overline{N}_{ij}}$ denotes the reports from individuals outside $\overline{N}_{ij}$. Clearly, t_{ij} respects the measurability constraints.

Let $m_i^s:\mathbb{R}^{\#\left(\overline{N}_i\right)}\to\mathbb{R}$ denote individual i's reporting strategy. Given the risk sharing arrangement t specified above, it is easy to see that, after endowment realizations, it is a Nash equilibrium for each individual to report his own e_i truthfully, i.e., setting $m^s(e)\equiv e$. This is because, given the endowment realizations e and the induced local state I_{ij} for each $j\in N_i$, strategically individual i should choose m_i to maximize his final consumption under t:

$$x_i^t(e,m) = e_i - \sum_{j \in N_i} t_{ij}(e,m)$$
$$= \left[e_i - \sum_{j \in N_i} \tilde{t}_{ij}\left(I_{ij}, m_{N \setminus \overline{N}_{ij}}\right) + \sum_{j \in N_i} |e_j - m_j| \right] - d_i |e_i - m_i|,$$

which depends on m_i only via the last term, $-d_i |e_i - m_i|$. It is then a dominant strategy for individual i to set $m_i = e_i$.

Anticipating this global truthful revelation of endowment realizations, it is obvious that $\tilde{t}$ should be configured to implement the global equal sharing rule[53], with the understanding that $m_{N \setminus \overline{N}_{ij}} = e_{N \setminus \overline{N}_{ij}}$ in equilibrium ex post.

In summary, global communication as specified above completely solves all information problems, and effectively (in the sense of ex post dominant strategy implementability) produces an informational network G' that is given by the complete network. Then, the local Borch rule, applied to the complete network G' (or the corresponding contractibility structure Q as defined in A.1), immediately implies that the first best global equal sharing can be achieved.

(b) Local Announcements

With local announcement, the effective (ex post dominant strategy implementable) informational network G' is effectively given by connecting all 2nd-order neighbors in the physical network G.

Let x'^* denote the constrained Pareto efficient consumption plan computed according to Proposition 4 with the informational network G'. As G is connected, there exists a profile of bilateral transfer rules $\tilde{t}$ defined on physical transfer links in G that satisfies the following two conditions:

$\tilde{t}$ induces the consumption plan x'^*; and

for every $i \in N$, individual i's exposure to e_k for any individual k of distance 2 to individual i in G is implemented by $\tilde{t}$ completely through a shortest path between i and k in G.

Notice that the existence of such a transfer arrangement $\tilde{t}$ is guaranteed by the complete arbitrariness in the configuration of superfluous cyclical transfers.

The key implication of condition ii) is that, for any linked pair ij in G, whenever there exists some individual k of distance 2 to *both i and j*, then the net share β_{ijk} of e_k transferred from i to j must be exactly 0. This is because, either i's or j's exposure of e_k should be completely channeled via their shortest paths (of length 2) to individual k, which necessarily does not

[53] This is clearly feasible under connectedness of G.

include link ij; moreover, any other individual's shortest path to k does not include link ij, either. Similarly, another implication of condition $ii)$ is that, for any lined pair ij in G and any $k \in \overline{N}_{ij}$, between ij there is zero share of e_k being transferred, i.e., $\beta_{ijk} = 0$.

With the two implications of condition $ii)$ in mind, we deduce that, as $\tilde{t}_{ij}$ can be contingent on endowment realizations of individuals in

$$\overline{N}_{ij}^{'} = \overline{N}_{ij} \cup \left(N_i \backslash \overline{N}_j\right) \cup \left(N_j \backslash \overline{N}_i\right) \cup \{k : k \text{ is of distance 2 to both } i \text{ and } j\},$$

i.e., the common neighborhood of ij under the supergraph $G^{'}$ that treats distance-2 individuals in G as linked (in $G^{'}$), the transfer arrangement $\tilde{t}$ admits the following linear representation:

$$\tilde{t}_{ij}(e) = \tilde{\alpha}_{ij} e_i - \tilde{\alpha}_{ji} e_j + \sum_{k \in N_i \backslash \overline{N}_j} \tilde{\beta}_{ijk} e_k - \sum_{k \in N_j \backslash \overline{N}_i} \tilde{\beta}_{ijk} e_k.$$

We now proceed to construct a risk sharing arrangement t using ex post messages based on $\tilde{t}$.

This can be achieved by letting each individual i submit a report m_i of all endowment realizations i observes, i.e., $(e_k)_{k \in \overline{N}_i}$. As m_i is observed by i's neighbors, the bilateral transfer contract t_{ij} between i and i's neighbor j can be contingent on I_{ij} as well as (m_i, m_j).

Consider the following specification of t_{ij}:

$$\begin{aligned} t_{ij}(I_{ij}, m_i, m_j) &= \tilde{t}_{ij}\left(I_{ij}, (m_{ik})_{k \in N_i \backslash \overline{N}_j}, (m_{jk})_{k \in N_j \backslash \overline{N}_i}\right) + C\left(|e_j - m_{ij}| - |e_i - m_{ji}|\right) \\ &= \tilde{\alpha}_{ij} e_i - \tilde{\alpha}_{ji} e_j + \sum_{k \in N_i \backslash \overline{N}_j} \tilde{\beta}_{ijk} m_{ik} - \sum_{k \in N_j \backslash \overline{N}_i} \tilde{\beta}_{jik} m_{jk} \\ &\quad + C\left(|e_j - m_{ij}| - |e_i - m_{ji}|\right) \end{aligned}$$

where m_{ik} denotes individual i's report of e_k and C is constant given by

$$C := \max_{ij \in G} \sum_{k \in N_i \backslash \overline{N}_j} \left|\tilde{\beta}_{ikj}\right|. \tag{36}$$

Again, i's final consumption under t is given by

$$\begin{aligned} x_i^t(e, m) = &\left(1 - \sum_{j \in N_i} \tilde{\alpha}_{ij}\right) e_i + \sum_{j \in N_i} \left(\tilde{\alpha}_{ji} e_j - C|e_j - m_{ij}| + C|e_i - m_{ji}|\right) \\ &- \sum_{j \in N_i} \left(\sum_{k \in N_i \backslash \overline{N}_j} \tilde{\beta}_{ijk} m_{ik} - \sum_{k \in N_j \backslash \overline{N}_i} \tilde{\beta}_{jik} m_{ji}\right) \end{aligned}$$

$$= \left[\left(1 - \sum_{j\in N_i} \tilde{\alpha}_{ij}\right) e_i + \sum_{j\in N_i} \left(\tilde{\alpha}_{ji} e_j + C\left|e_i - m_{ji}\right| + \sum_{k\in N_j \setminus \overline{N}_i} \tilde{\beta}_{jik} m_{ji} \right) \right]$$
$$- \sum_{j\in N_i} \left(C\left|e_j - m_{ij}\right| + \sum_{k\in N_i\setminus \overline{N}_j} \tilde{\beta}_{ikj} m_{ij} \right),$$

which by (36) strictly increases in m_{ij} whenever $m_{ij} < e_j$, and strictly decreases in m_{ij} whenever $m_{ij} > e_j$. Hence, ex post individual i's dominant strategy is to set $m_{ij} = e_j$.

Lastly, notice that no information about third-order neighbors can be possibly transmitted under the protocol of local announcements.

In summary, all individuals will truthfully report $(e_k)_{k\in\overline{N}_i}$ in an ex post dominant strategy equilibrium, together achieving the constrained Pareto efficient consumption plan with respect to the augmented informational network $G^{'}$, a supergraph of the physical transfer network G that links all distance-2 neighbors in G.

(c) Local Comments

With local comments, the effective (ex post dominant strategy implementable) informational network $G^{'}$ is effectively given by connecting all neighbors within a distance of 3 in the physical network G.

Now, each individual i may submit to each neighbor $j \in N_i$ a report m_{ij} of the endowment realizations i observe, i.e., $I_i \equiv (e_k)_{k\in\overline{N}_i}$. Specifically, $m_{ij} \in \mathbb{R}^{|\overline{N}_i|}$ and we write m_{ijk} to denote i's report of e_k to individual j.

Now that for any linked pair ij, they can commonly observe endowment realizations I_{ij}, all comments received by i and all comments received by j (say, displayed on i's and j's Facebook pages). This facilitates transmission of distance-3 information: for any path $i-j-k-h$ in G, individual i can now observe on j's comment book a comment left by k about h's endowment realization e_h. We now simply need to properly construct the bilateral contracts to incentivize truthful reporting ex post.

A formal procedure can be constructed in a similar way to the procedure for (b) "local announcements" above. For avoid repetition, we now just provide a description of the key idea.

If i lies about own endowment realization e_i to $j \in N_i$, this is immediately detectable by j. Then by properly specifying a punishment transfer from i to j based on $|e_i - m_{iji}|$, we can incentivize i to truthfully reports his own endowment realizations to his neighbors. This implies that, each individual i can now observe a truthful report of his 2nd-order neighbors, based on their truthful comments sent to i's first-order neighbors.

Now consider a linked pair ij. If i lies to j about e_k for some $k \in N_i \backslash \overline{N}_j$, this is detectable by j because j also observes k's report to i, namely m_{ki}, which includes a truthful report m_{kik} of e_k. Contract t_{ij} may then specify a sufficient punishment transfer from i to j, which ensures the truthfulness of m_{ijk} about e_k in a Nash equilibrium. Hence, each individual i can now observe a truthful report of his 3rd-order neighbor's endowment e_k, which is included in a report from one of i's 2nd-order neighbor to one of i's (1st-order) neighbor.

Now, suppose that both i and j "effectively know" e_k for some $k \notin \overline{N}_{ij}$. If $k \in N_i \cup N_j$, then k submits a truthful report to either i or j, which is commonly observable by i and j, so t_{ij} can be optimally contingent on m_{kik} or m_{kjk}. If $k \notin N_i \cup N_j$, there are two possibilities.

First, if k is a 2nd-order neighbor of i (or j with similar arguments), then there must exist some $h \in N_i$ that submits a report m_{hi} to i, which is commonly observed by ij and includes a truthful report of e_k, so t_{ij} can be optimally contingent on m_{hik}.

Second, if k is a 3rd-order neighbor of both i and j, there are three sub-cases. In sub-case 1, ij are both linked to h, of whom k is a 2nd-order neighbor. Then ij commonly observe a report received by h, which includes a truthful report of e_k. In sub-case 2, there exists a path $k \to i$ and a path $k \to j$ that both pass through some $h \in N_k$, but the condition for sub-case 1 does not hold. Then there is no report of e_k that is commonly observed by ij, but h is a diagonal node for link ij in a pentagon subgraph. This does not affect risk-sharing efficiency due to the redundancy of link ij: efficient exposure to e_k can be channeled completely through the two paths from h to i and j respectively, and it has been shown in the above arguments that e_k or a truthful report of e_k is commonly observable by the two contracting parties in each link along the two paths. In sub-case 3, any two paths $k \to i$ and $k \to j$ must be disjoint (except at k), in which case k is the diagonal node to link ij in a heptagon subgraph. Again this does not affect risk-sharing efficiency due to the redundancy of link ij: efficient exposures to e_k can still be channeled completely through the two paths. In particular consider the path $k - i_2 - i_1 - i$, and notice that a truthful report of e_k from i_2 to i_1 is commonly observable by both i_1 and i, thus being contractible.

This completes the proof that "local comment" leads to an implementable informational network with effective local observability of 2nd-order and 3rd-order neighbors in the physical network G. Lastly, notice that no information about 4th-order neighbors is possible with local comment, so no other contracts can do better.

(d) Private Communication

With private communication, the informational network remains unchanged. This is because when messages are completely private there is no information spillover to any other party. In the meanwhile, the ex post messaging game is a zero-sum game (as the messages are mapped into net transfers). Hence, given each local state I_{ij}, both i and j are guaranteed the value of the ex post game in any Nash equilibrium, so the equilibrium payoffs do not depend on nonlocal endowment realizations. Thus the implementable informational network remains unchanged. ∎

C.13 Detailed Specification and Proof for Proposition 12

In our setting, for a given network G, individual i's Myerson value is defined by

$$MV_i(G) := \sum_{S \subseteq N} \frac{(\#(S)-1)(n-\#(S))}{n!} \cdot \frac{1}{2} r\sigma^2 \left[TVar\left(G|_{(S\setminus\{i\})}\right) + \sigma^2 - TVar\left(G|_S\right)\right]$$

where $\#(S)$ denotes the number of individuals in a subset S of N, and $G|_S$ denotes the subgraph of G restricted to the subset S of individuals. Given the CARA-normal specification, $TVar\left(G|_{(S\setminus\{i\})}\right) + \sigma^2 - TVar\left(G|_S\right)$ is the surplus from risk reduction through i's links in S.

Notice that, given any $S \subseteq N$,

$$TVar\left(G|_{S\setminus\{i\}}\right) - TVar\left(G|_S\right) = 1 - \frac{1}{d_i\left(G|_S\right)+1} + \sum_{k \in N_i\left(G|_S\right)} \frac{1}{d_k\left(G|_S\right)\left[d_k\left(G|_S\right)+1\right]},$$

which is strictly increasing in $d_i\left(G|_S\right)$ but strictly decreasing in $d_k\left(G|_S\right)$ for each $j \in N_k\left(G|_S\right)$. Moreover, for any $k \in N$, $d_k\left(G|_S\right)$ is weakly increasing in S, i.e., $S \subseteq S' \Rightarrow d_k\left(G|_S\right) \leq d_k\left(G|_{S'}\right)$.

Consider any pairwise stable network G under the Myerson-value transfers. Then, if i, j are linked, it must be that

$$MV_i(G) - MV_i(G - ij) \geq c.$$

Fixing ij, for each $S \subseteq N$, we have

$$\begin{aligned} & TVar\left(G - ij|_{S\setminus\{i\}}\right) - TVar\left(G - ij|_S\right) \\ = & \begin{cases} TVar\left(G|_{S\setminus\{i\}}\right) - TVar\left(G|_S\right), & \text{if } j \notin S \\ 1 - \frac{1}{d_i\left(G|_S\right)} + \sum_{k \in N_i\left(G|_S\right)\setminus\{j\}} \frac{1}{d_k\left(G|_S\right)\left[d_k\left(G|_S\right)+1\right]}, & \text{if } j \in S \end{cases} \end{aligned}$$

so

$$\left[TVar\left(G|_{S\setminus\{i\}}\right) - TVar\left(G|_S\right)\right] - \left[TVar\left(G - ij|_{S\setminus\{i\}}\right) - TVar\left(G - ij|_S\right)\right]$$

$$
\begin{aligned}
&= \mathbb{1}\{j \in S\} \cdot \left[\frac{1}{d_i(G|_S)[d_i(G|_S)+1]} + \frac{1}{d_j(G|_S)[d_j(G|_S)+1]} \right] \\
&\geq \mathbb{1}\{j \in S\} \cdot \left[\frac{1}{d_i(G)[d_i(G)+1]} + \frac{1}{d_j(G)[d_j(G)+1]} \right]
\end{aligned}
$$

Averaging over all possible $S \subseteq N$, we get

$$
MV_i(G) - MV_i(G - ij) \geq \frac{1}{2} \cdot \left[\frac{1}{d_i(G)[d_i(G)+1]} + \frac{1}{d_j(G)[d_j(G)+1]} \right]
$$

as

$$
\sum_{S \subseteq N} \frac{(\#(S)-1)(n-\#(S))}{n!} \mathbb{1}\{j \in S\} = Pr\{i \text{ arrives later than } j\} = \frac{1}{2}.
$$

From the perspective of social efficiency, the link ij in G is (strictly) socially efficient if

$$
\frac{1}{d_i(G)[d_i(G)+1]} + \frac{1}{d_j(G)[d_j(G)+1]} > 2c.
$$

Thus we can conclude that, given any pairwise stable network G under the Myerson-value transfers, whenever a link ij is (strictly) socially efficient, it will be present in G, because the increments in both i's and j's private benefits strictly exceed the cost of linking c:

$$
\begin{aligned}
MV_i(G) - MV_i(G - ij) &> \frac{1}{2} \cdot 2c = c \\
MV_j(G) - MV_j(G - ij) &> \frac{1}{2} \cdot 2c = c.
\end{aligned}
$$